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Nonlinear state estimation in flexible-link multibody systems through reduced-order models

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ABSTRACT — *Performances of flexible link multibody systems, in terms of accuracy and repeatability, can be negatively affected by link flexibility that causes unwanted vibration. Therefore, advanced controllers are necessary for vibration suppression. The synthesis of such controllers typically relies on the knowledge of all the system state variables, whose direct measurement is complicated, or at least of a meaningful set of the most relevant ones. For such a reason, these state variables should be estimated through state observers. In particular, state observers based on reduced-order dynamic models should be employed, to reduce the computational effort. This paper shows some preliminary results on state estimation in flexible-link multibody systems based on nonlinear, reduced-order dynamic model formulated through independent coordinates. Reduction is performed through a modified Craig-Bampton strategy. Numerical simulations of a six-bar planar mechanism show that the proposed observer delivers accurate estimates of both the rigid and elastic variables.*

1 Introduction

In the last years, the use of flexible link multibody systems (FLMBs) is boosted by a growing sensibility to sustainability and energy saving. The accuracy and the repeatability of these systems can be compromised by their flexibility that often causes unwanted vibration.

Therefore, the synthesis of effective control scheme is necessary for vibration suppression. Most of these controllers are typically model-based and require precise knowledge of all the system state variables (i.e. position and velocity of each coordinate), or at least to a meaningful set of them, to ensure effective control of the infinite number of degrees of freedom (dofs) that characterize such a kind of system. In practical application the state variables are rarely directly measurable [1]. State observers should hence be employed to estimate the unmeasured state variables that are of interest for control.

Nevertheless, observers typically require accurate system models, and in the case of FLMBs, models are either complex and cumbersome. In order to address this challenging task, observers based on linearized models have been developed [2]. Many contributions can be found also about the state estimation for a single flexible link [3]-[6]. However, for multiple links, the problem becomes more severe.

Usually, the state variables that are of interest for control purpose are a slightly smaller subset of the whole state vector employed at the modeling stage, and are those having prominent observability and controllability [2]. Therefore, state observers based on effective reduced order dynamic models should be employed to ensure reliable estimation of the most important state variables while reducing the computational effort.

This paper shows some preliminary results on state estimation in FLMBSs through nonlinear reduced order dynamic models formulated through independent coordinates (and hence through a set of ordinary differential equations). Model reduction is performed by means of a modified Craig-Bampton strategy [7], which allows keeping to a minimum the size of the nonlinear dynamic model of a FLMBS while preserving its accuracy in a given frequency range even in the presence of large displacements. The capability of the proposed observer based on the Extended Kalman Filter (EKF) algorithm [8] to deliver accurate estimates of both the gross and fine motion of a FMLBS is proved by means of numerical simulations.

2 Modeling

State estimation can be effectively carried out if the flexible-link manipulator is adequately modeled. Many approaches to the dynamic modeling of flexible manipulators have been proposed in literature [9]-[12], including Finite Element Method (FEM), lumped parameter approximations, and assumed modes methods.
In this paper a dynamic model of FLMBSs formulated through Ordinary Differential Equations (ODEs) and based on the Equivalent Rigid-Link (ERLS) approach [12] is adopted and briefly recalled in the following Section.

2.1 Full-order dynamic model: ERLS approach

The ERLS approach requires the use of FEM to get a dynamic model of a FLMBS with a finite number of degrees of freedom. Following such an approach the total motion of the system is notationally splitted into the large rigid-body motion of a rigid-link moving reference configuration (the ERLS) and the small elastic deformation of the flexible links with respect to the ERLS itself. Basically the links of the mechanism are divided into finite elements and the position of each node of an element is computed as the sum of the position taken by the node on the rigid-body reference mechanism and the nodal elastic displacement with respect to the reference mechanism.
The equations of motion, obtained by expressing the dynamic equilibrium of the system through the principle of virtual work, are given by the following set of nonlinear ODEs:

$$\begin{bmatrix} \mathbf{M}(\mathbf{q}) & \mathbf{M}(\mathbf{q})\mathbf{S}(\mathbf{q}) \\ \mathbf{S}^{\mathrm{T}}(\mathbf{q})\mathbf{M}(\mathbf{q}) & \mathbf{S}^{\mathrm{T}}(\mathbf{q})\mathbf{M}(\mathbf{q})\mathbf{S}(\mathbf{q}) \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{q}} \end{Bmatrix} + \begin{bmatrix} 2\mathbf{M}_G(\mathbf{q}) + \mathbf{C}(\mathbf{q}) & \mathbf{M}(\mathbf{q})\dot{\mathbf{S}}(\mathbf{q},\dot{\mathbf{q}}) \\ \mathbf{S}^{\mathrm{T}}(\mathbf{q})\left(2\mathbf{M}_G(\mathbf{q}) + \mathbf{C}(\mathbf{q})\right) & \mathbf{S}^{\mathrm{T}}(\mathbf{q})\mathbf{M}(\mathbf{q})\dot{\mathbf{S}}(\mathbf{q},\dot{\mathbf{q}}) \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{q}} \end{Bmatrix} + \\ + \begin{bmatrix} \mathbf{K}(\mathbf{q}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{q} \end{Bmatrix} = \begin{bmatrix} \mathbf{M}(\mathbf{q}) & \mathbf{I} \\ \mathbf{S}^{\mathrm{T}}(\mathbf{q})\mathbf{M}(\mathbf{q}) & \mathbf{S}^{\mathrm{T}}(\mathbf{q}) \end{bmatrix} \begin{Bmatrix} \mathbf{g}(\mathbf{q}) \\ \mathbf{f}(\mathbf{q}) \end{Bmatrix} \tag{1}$$

Where $\mathbf{q}$ is the vector of the generalized coordinates of the ERLS and $\mathbf{u}$ is the vector of the elastic displacements of all the nodes of the finite elements. in Eq. (1) $\mathbf{M}$, $\mathbf{K}$, $\mathbf{C}$, and $\mathbf{M}_G$ are the mass, stiffness, damping, and centrifugal and Coriolis matrices, respectively, obtained by assembling the consistent matrices of the finite elements; $\mathbf{g}$ is the gravity acceleration vector, and $\mathbf{f}$ is the external force vector; $\mathbf{S}$ and $\dot{\mathbf{S}}$ are the ERLS sensitivity coefficient matrix for all the nodes and its time derivative, respectively (the kinematics of the ERLS is defined according to the ordinary rules holding for a chain of rigid links).

2.2 Reduced-order dynamic model: modified Craig-Bampton method

In order to reduce the computational effort related to the estimation process, model size should be kept as small as possible. For this reason the motion equation in Eq (1) is reduced by means of the reduction strategy proposed in [7], and here brefly recalled.

Let us partition the displacement vector $\mathbf{x}^{\mathrm{T}} = \left\{\mathbf{q}^{\mathrm{T}} \quad \mathbf{u}^{\mathrm{T}}\right\} \in \mathbb{R}^n$ of Eq (1) into two subvectors: $\mathbf{x}_m^{\mathrm{T}} = \left\{\mathbf{q}^{\mathrm{T}} \quad \mathbf{u}_m^{\mathbf{T}}\right\} \in \mathbb{R}^m$ and $\mathbf{x}_s^{\mathrm{T}} = \left\{\mathbf{u}_s^{\mathbf{T}}\right\} \in \mathbb{R}^s$, namely master and slave dofs, respectively (m+s=n). The master subvector comprises all the ERLS coordinates and possibly some meaningful elastic coordinates, such as those nodes where external forces are applied or sensors are located. Let us partition accordingly the motion equations and rewrite them in a more compact form:

$$\bar{\mathbf{M}}(\mathbf{q})\begin{Bmatrix}\ddot{\mathbf{x}}_m \\ \ddot{\mathbf{x}}_s\end{Bmatrix}+\bar{\mathbf{C}}(\mathbf{q},\dot{\mathbf{q}})\begin{Bmatrix}\dot{\mathbf{x}}_m \\ \dot{\mathbf{x}}_s\end{Bmatrix}+\bar{\mathbf{K}}(\mathbf{q})\begin{Bmatrix}\mathbf{x}_m \\ \mathbf{x}_s\end{Bmatrix}=\bar{\mathbf{F}}(\mathbf{q}) \tag{2}$$

The full-order set of coordinates $\mathbf{x}$ can be reduced to a smaller one $\mathbf{p}$ by means of the configuration-dependent Craig-Bampton reduction matrix $\mathbf{H}(\mathbf{q},\mathbf{q}^*)$:

$$\begin{gathered}\mathbf{x}=\mathbf{H}(\mathbf{q},\mathbf{q}^*)\mathbf{p} \\ \mathbf{x}=\begin{Bmatrix}\mathbf{x}_m \\ \mathbf{u}_s\end{Bmatrix}\in\mathbb{R}^{n=m+s};\ \mathbf{p}=\begin{Bmatrix}\mathbf{x}_m \\ \boldsymbol{\eta}\end{Bmatrix}\in\mathbb{R}^{m+p} \\ p\ll s\end{gathered} \tag{3}$$

where $\boldsymbol{\eta}$ is the vector of the retained interior modal coordinates, while $\mathbf{H}$ is defined as follows:

$$\mathbf{H}(\mathbf{q},\mathbf{q}^*)=\begin{bmatrix}\mathbf{I} & \mathbf{0} \\ \mathbf{B}(\mathbf{q}) & \tilde{\boldsymbol{\Psi}}(\mathbf{q},\mathbf{q}^*)\end{bmatrix} \tag{4}$$

In Eq. (4) $\mathbf{I}$ and $\mathbf{0}$ are the identity and null matrices, respectively; $\mathbf{B}=\mathbf{B}(\mathbf{q})$ and $\tilde{\boldsymbol{\Psi}}(\mathbf{q},\mathbf{q}^*)$ are the configuration-dependent Guyan's and interior mode matrices, respectively; $\mathbf{q}^*$ is the equilibrium configuration at which the interior modes are computed.
The reduced model is obtained by pre-multiplying Eq. (2) by $\mathbf{H}^{\mathrm{T}}$ and by substituting Eq. (3) in it:

$$\begin{gathered}\underbrace{\mathbf{H}^{\mathrm{T}}(\mathbf{q},\mathbf{q}^*)\bar{\mathbf{M}}(\mathbf{q})\mathbf{H}(\mathbf{q},\mathbf{q}^*)}_{\tilde{\mathbf{M}}}\begin{Bmatrix}\ddot{\mathbf{x}}_m \\ \ddot{\boldsymbol{\eta}}\end{Bmatrix}+\underbrace{\mathbf{H}^{\mathrm{T}}(\mathbf{q},\mathbf{q}^*)\bar{\mathbf{C}}(\mathbf{q},\dot{\mathbf{q}})\mathbf{H}(\mathbf{q},\mathbf{q}^*)}_{\tilde{\mathbf{C}}}\begin{Bmatrix}\dot{\mathbf{x}}_m \\ \dot{\boldsymbol{\eta}}\end{Bmatrix}+ \\ +\underbrace{\mathbf{H}^{\mathrm{T}}(\mathbf{q},\mathbf{q}^*)\bar{\mathbf{K}}(\mathbf{q})\mathbf{H}(\mathbf{q},\mathbf{q}^*)}_{\tilde{\mathbf{K}}}\begin{Bmatrix}\mathbf{x}_m \\ \boldsymbol{\eta}\end{Bmatrix}=\underbrace{\mathbf{H}^{\mathrm{T}}(\mathbf{q},\mathbf{q}^*)\bar{\mathbf{F}}(\mathbf{q})}_{\tilde{\mathbf{F}}}\end{gathered} \tag{5}$$

3 State estimation

The synthesis of an observer requires to convert the dynamic model in Eq. (5) to a state-space form, as follows:

$$\begin{cases}\dot{\mathbf{z}}=\boldsymbol{f}(\mathbf{z},\tilde{\mathbf{F}})=\begin{Bmatrix}\dot{\mathbf{p}} \\ \tilde{\mathbf{M}}^{-1}\left(\tilde{\mathbf{F}}-\tilde{\mathbf{C}}\dot{\mathbf{p}}-\tilde{\mathbf{K}}\mathbf{p}\right)\end{Bmatrix} \\ \mathbf{y}=\boldsymbol{g}(\mathbf{z},\tilde{\mathbf{F}})\end{cases} \tag{6}$$

where $\mathbf{z}=\left\{\mathbf{p}^{\mathrm{T}}\quad\dot{\mathbf{p}}^{\mathrm{T}}\right\}^{\mathrm{T}}$ is the state vector and $\dot{\mathbf{z}}$ its derivative, $\mathbf{y}$ is the vector of the measured system outputs. $\tilde{\mathbf{F}}$ is the reduced-order vector of the forces acting on the system, and is the input of the state space representation. In the most general case both the system equation $\boldsymbol{f}$ and the measurement equation $\boldsymbol{g}$ in Eq. (6) are nonlinear function of the state.

3.1 Extended Kalman Filter

The estimation algorithm employed is the EKF [8], the simplest extension of the well-known Kalman filter to nonlinear state estimation. In its widespread formulation the EKF is a discrete recursive algorithm and requires a discrete and stochastic system model representation:

$$\begin{cases} \mathbf{z}_k = \boldsymbol{f}_d(\mathbf{z}_{k-1}, \tilde{\mathbf{F}}_{k-1}, \mathbf{v}_{k-1}) \\ \mathbf{y}_k = \boldsymbol{g}(\mathbf{z}_k, \mathbf{w}_k) \end{cases} \tag{7}$$

where $\mathbf{v}_{k-1}$ and $\mathbf{w}_k$ are the system process noise and measurement noise, $\boldsymbol{f}_d$ is the discrete time state equation, *k* refers to the *k*th time sample.

Starting from a given initial values for the state $\hat{\mathbf{z}}_0$ and its covariace $\hat{\mathbf{P}}_0^{\mathbf{zz}}$, the EKF perform recursively the state estimation, as follows:

- PREDICTION

  State prediction by means of the nonlinear discrete system equation

  $\hat{\mathbf{z}}_{k|k-1} = \boldsymbol{f}_d\left(\hat{\mathbf{z}}_{k-1|}, \tilde{\mathbf{F}}_{k-1}, \mathbf{v}_{k-1}\right)$

  State covariance prediction, by means of the Jacobian of the system equation

  $\hat{\mathbf{A}}_{k-1} = \partial \boldsymbol{f} / \partial \mathbf{z}\big|_{\hat{\mathbf{z}}_{k-1}, \tilde{\mathbf{F}}_{k-1}}$

  $\hat{\mathbf{P}}_{k|k-1}^{\mathbf{zz}} = \hat{\mathbf{A}}_{k-1} \hat{\mathbf{P}}_{k-1}^{\mathbf{zz}} \hat{\mathbf{A}}_{k-1}^{\mathrm{T}} + \mathbf{Q}$

  Observation prediction by means of the nonlinear measurement equation

  $\hat{\mathbf{y}}_{k|k-1} = \boldsymbol{g}\left(\hat{\mathbf{z}}_{k|k-1}, \mathbf{w}_k\right)$

  Observation covariance prediction, by means of the Jacobian of the measurement equation

  $\hat{\mathbf{D}}_{k|k-1} = \partial \boldsymbol{g} / \partial \mathbf{z}\big|_{\hat{\mathbf{z}}_{k|k-1}}$

  $\hat{\mathbf{P}}_{k|k-1}^{\mathbf{yy}} = \hat{\mathbf{D}}_{k|k-1} \hat{\mathbf{P}}_{k|k-1}^{\mathbf{zz}} \hat{\mathbf{D}}_{k|k-1}^{\mathrm{T}} + \mathbf{R}$

  $\hat{\mathbf{P}}_{k|k-1}^{\mathbf{zy}} = \hat{\mathbf{P}}_{k|k-1}^{\mathbf{zz}} \hat{\mathbf{D}}_{k|k-1}^{\mathrm{T}}$

- CORRECTION

  Kalman gain computation

  $\mathbf{K}_k = \hat{\mathbf{P}}_{k|k-1}^{\mathbf{zy}} \left[\hat{\mathbf{P}}_{k|k-1}^{\mathbf{yy}}\right]^{-1}$

  State correction by means of the Kalman gain and the measured output variables $\mathbf{y}_k$

  $\hat{\mathbf{z}}_k = \hat{\mathbf{z}}_{k|k-1} + \mathbf{K}_k\left(\mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1}\right)$

  State covariance correction

  $\hat{\mathbf{P}}_k^{\mathbf{zz}} = \hat{\mathbf{P}}_{k|k-1}^{\mathbf{zz}} - \mathbf{K}_k \hat{\mathbf{P}}_{k|k-1}^{\mathbf{yy}} \mathbf{K}_k^{\mathrm{T}}$

- NEW ITERATION INPUT

  $\hat{\mathbf{z}}_k$, $\hat{\mathbf{P}}_k^{\mathbf{zz}}$

- ESTIMATED STATE AT TIME k

  $\hat{\mathbf{z}}_k$

Q and **R** are the covariance matrices of the model and measurement noises, respectively, in practice these matrices are employed to compute the gain $\mathbf{K}_k$ of the filter correction. In order to properly synthesize the filter, **R** should be measured while the process noise covariance tuned to give the appropriate weights to model predictions and noisy measurements in state estimates.

4 Observer validation

The planar mechanism shown in Fig. 1 and lying on the horizontal plane is assumed as the test case for the validation of the nonlinear observed based on a nonlinear reduced-order dynamic model of FLMBS. Three motors are supposed to drive links 1, 2 and 4-5. The links have circular cross section and dimensions as shown in Fig. 1(a). All the links are supposed to be made of Aluminium with elastic modulus 69 GPa and mass density 2700 kg/m^3. They are modeled by uniform two-node and six-dof beam elements, the finite element model adopted is shown in Fig. 1(a). Lumped masses and inertias are used to account for the moving joints, and for the brakes and the rotors of the motors, as indicated in Tab. 1.

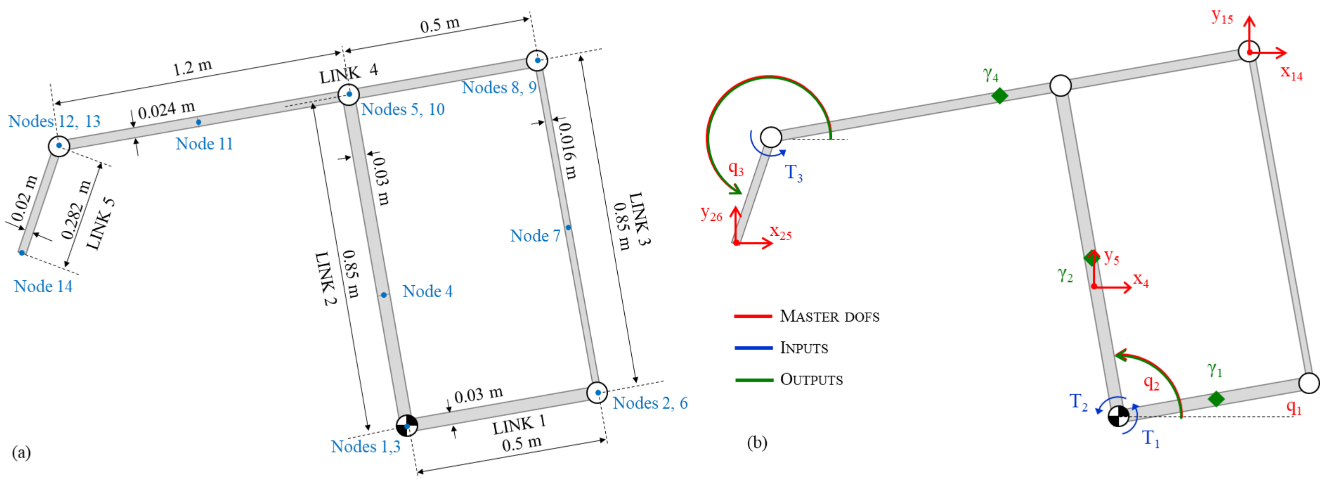


Fig. 1: Studied mechanism: finite element model (a), variables involved in the estimation process (b)

Node	Mass [kg]	Inertia [10^{-2}kgm^2]
1	7.644	1.30
2	0.392	0.00
3	9.517	2.29
4	0.400	0.00
5	0.648	0.00
6	0.671	0.00
8	0.383	0.00
9	0.658	0.00
10	0.308	0.00
12	1.537	0.00
13	0.095	0.15
14	0.046	0.00

Tab. 1: Lumped masses and inertia

The observer is synthesized through the EKF algorithm and the reduced order model presented in [7], where a 30-dof model has been obtained through the ERLS approach. Such a full order model has been then reduced through the modified Craig-Bampton strategy leading to a nonlinear 13 dofs model, containing 9 physical coordinates (i.e. the master dofs shown in red in Fig.1(b)) and 4 interior modal coordinates:

$$\mathbf{x}_m = \{q_1 \quad q_2 \quad q_2 \quad x_4 \quad y_5 \quad x_{14} \quad y_{15} \quad x_{25} \quad y_{26}\}^T, \quad \boldsymbol{\eta} \in \mathbb{R}^{4x1} \tag{8}$$

The resulting state vector of the first-order model used for estimation has therefore 26 variables.
The torques exerted by the three motors driving the system (T_1, T_2, and T_3 in Fig.1(b)) have been used as the measured inputs of the state observer:

$$\tilde{\mathbf{F}} = \{T_1 \quad T_2 \quad T_3 \quad 0_4 \quad \dots \quad 0_{13}\}^T \tag{9}$$

Additionally, six sensed outputs have been employed as the model output adopted to compute the observer innovation: i.e. the angular positions of the three-actuated links (q_1, q_2, and q_3) and the curvatures (strains) of the midpoints of links 1, 2 and 4 (see Fig. 1(b)):

$$\mathbf{y} = \{q_1 \quad q_2 \quad q_3 \quad \gamma_1 \quad \gamma_2 \quad \gamma_4\}^T \tag{10}$$

Such a choice of the output variables results in a nonlinear measurement equation, indeed, only q_1, q_2, and q_3, are a linear combination of the state variables.
A thorough observability analysis has been computed employing a linearized observability analysis in the whole manipulator workspace and it has proved that the selected set of input and output variables guarantees system observability.

The validation of the observer is carried out by means of a simulated test in Matlab. The motion of the reference manipulator, which can be thought of as the "real mechanism", is simulated through the full-order model. In contrast, the observer is based on the nonlinear reduced-order manipulator model. The signals of the measured inputs and outputs fed to the observer are corrupted with noise. Such a test allows for an effective assessment of the observer outcomes by comparing all the manipulator state variables, including the elastic ones which cannot be measured experimentally, and by properly evaluating the impact of the model truncation in the estimation. A simulation lasting 3 seconds has been tested, which highlights that no drift affect the estimates, i.e the observer is stable. A multi-rate EKF observer has been implemented, in particular, the continuous motion equations in Eq. (6) have been discretized by using the fourth-order Runge-Kutta method and an integration step of 0.25 ms, while the measured signals have been update at 500 Hz. In particular, three sinusoidal torques have been simulated to drive the actuated links:

$$T_1 = 1.4\sin(2\pi 8t) \quad T_2 = 7\sin(2\pi 10t) \quad T_3 = 0.1\sin(2\pi 11t) \tag{11}$$

Gaussian noises have been added on all the three simulated torques with amplitudes of 0.20% of the full scale of, respectively, 15 Nm, 60 Nm, and 2 Nm torque meters. Successively, the signals have been digitized trough a 24-bit ADC with input range ±10 V. Portions of the simulated input and output signals are shown, respectively, in Figs. 2 and 3.

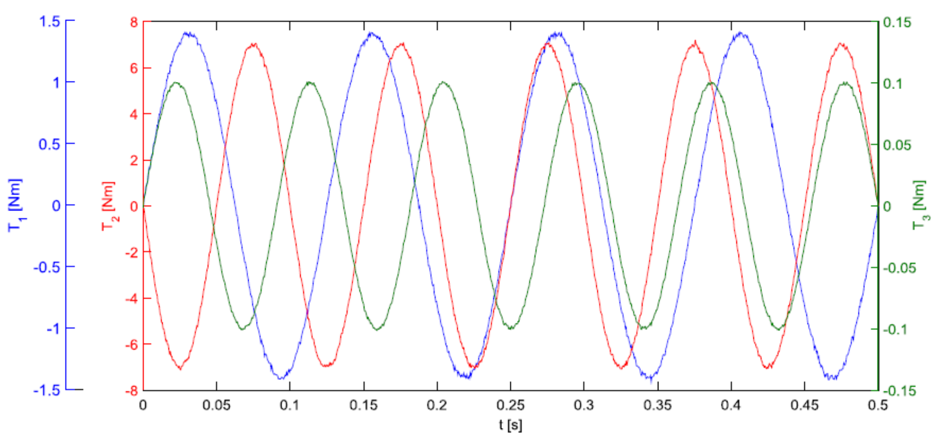


Fig. 2: Actuation forces fed as inputs to the observer

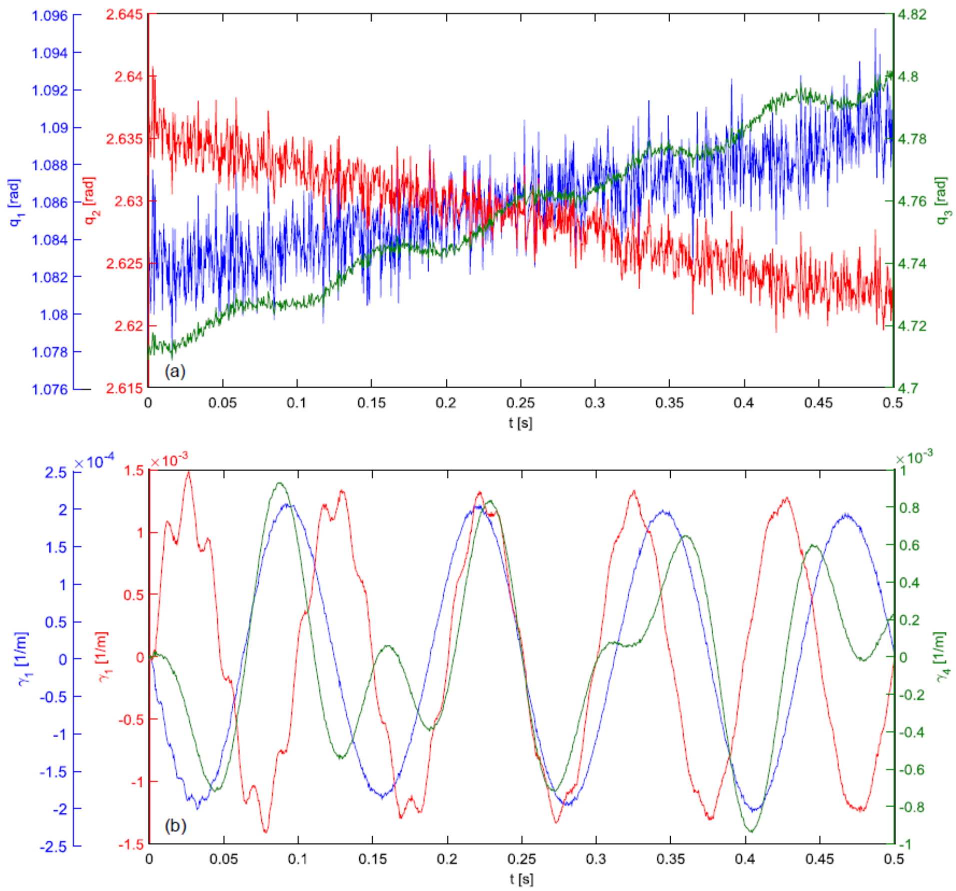
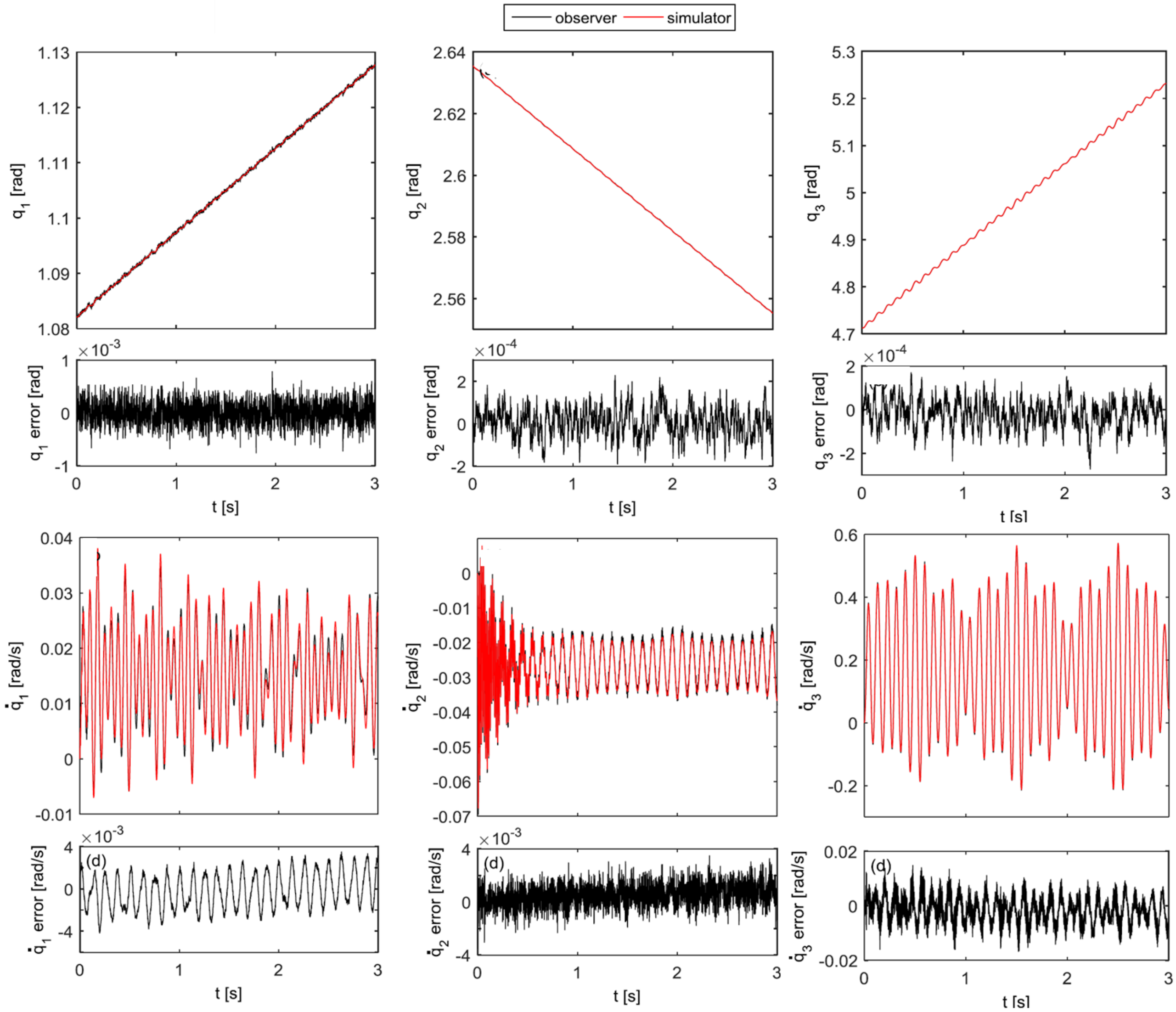


Fig. 3: Observer outputs: angular positions of the actuated links (a); curvatures of link 1, 2 and 4 (b)

The estimates of the angular positions and velocities of the three actuated links are plotted in Fig. 4. The same figure also shows the time-histories of the error between the actual variables (computed through the full-order model and free measurement error signals) and the estimated ones. Error diagrams clearly show that the observer is able to deliver accurate estimates of the manipulator gross motion.

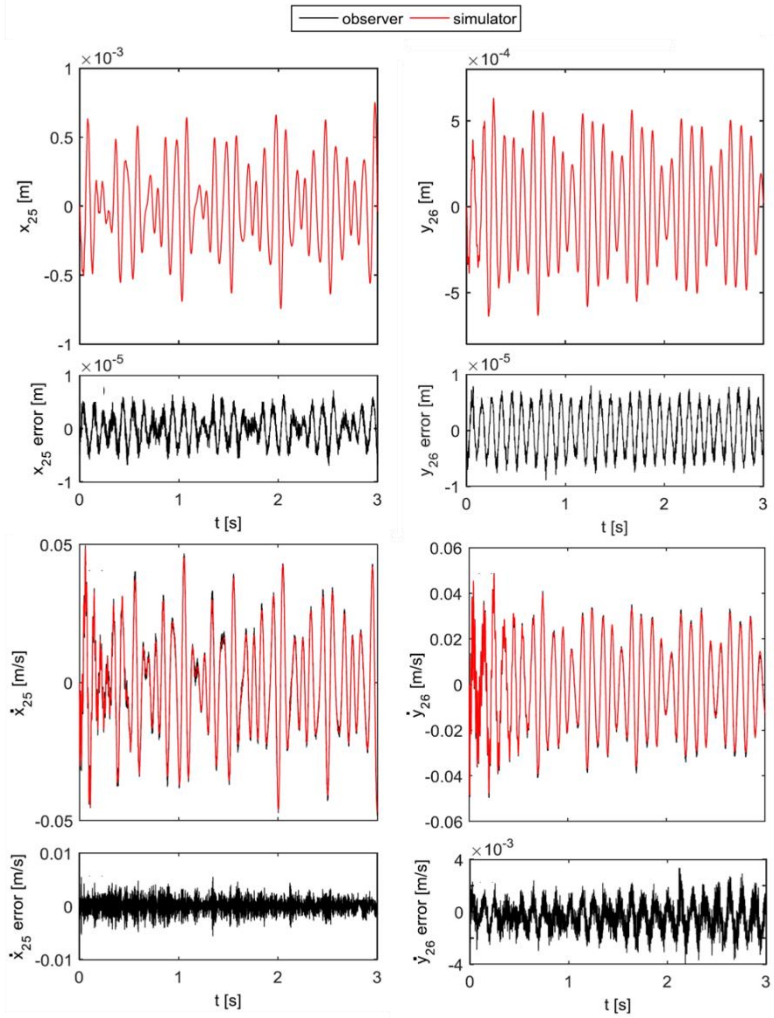
The observer capability of providing accurate estimates of the elastic state variables is proved by Fig. 5. This figure refers to linear displacements and velocities of the manipulator tip. Good agreement is confirmed to exist between the actual and the estimated variables, both in terms of amplitude and frequency content of the time-histories. In particular, further evidences of the observer effectiveness come from the fast Fourier transforms (FFT) of the elastic estimates, which are shown in Fig. 6. Indeed, the FFTs of the estimated elastic displacement variables are almost perfectly overlapped to the actual ones. Such a result also confirms the capability of the reduced-order model to represent correctly the dynamics of the manipulator, since not only are the excitation frequencies (8, 10, 11 Hz) matched correctly, which is rather intuitive and simple to achieve, but also the exact amplitudes of the harmonic components at the system natural frequencies 14.2 Hz and 67.0 Hz, which have been determined in [7]. Therefore, a very satisfactory agreement is proved to exist among both the estimated variables and the actual ones, as well as the reduced-order model and the full-order one.

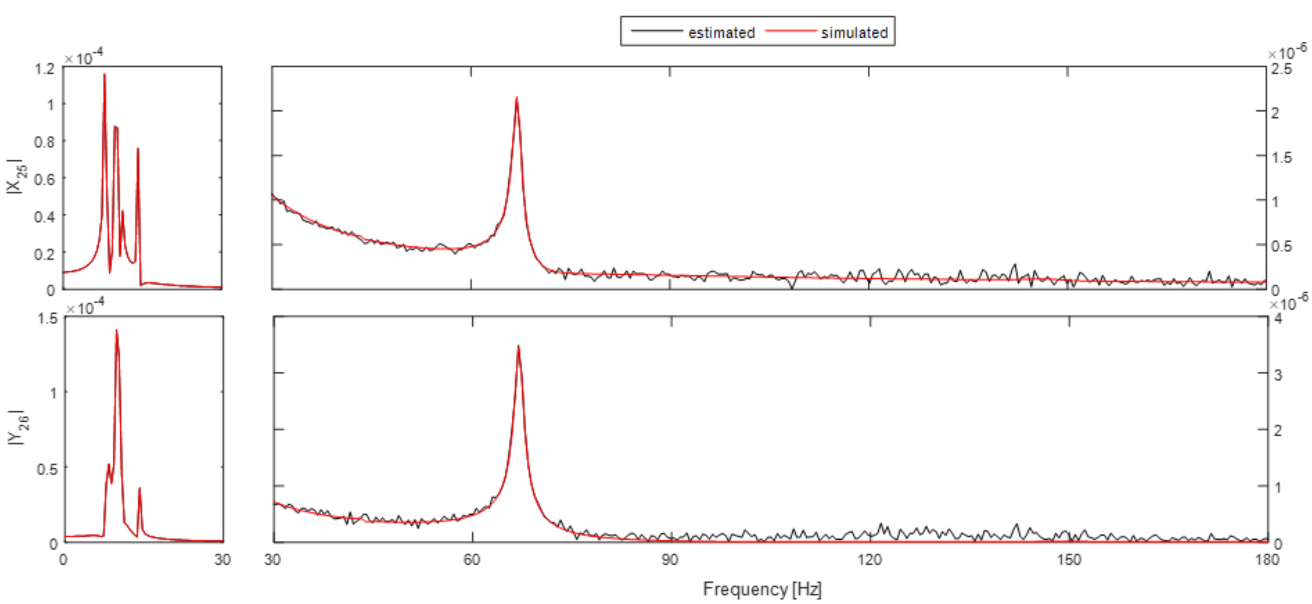
Finally, the comparison of the computational efforts related to the Matlab implementations of an observer based on the full-order dynamic model (60 state variables) and another one based on the reduced-order dynamic model (26 state variables) has shown that the latter reduces the computational time of 67%.



5 Conclusions

In this paper a state observer based on nonlinear reduced-order dynamic model for a representative flexible-link manipulator has been successfully synthesized. It has been proved that such an observer is able to deliver accurate estimates of both the rigid and elastic variables characterizing the motion of a rather complex FLMBS. The test also confirmed the reduced model capability to match correctly the multibody system dynamics in a given frequency range. The great reduction in time compared to an observer based on a full order dynamic model, makes the proposed observer promising for getting efficient state estimates in flexible-link multibody systems.


observer
simulator
x_{25} [m]
y_{26} [m]
x_{25} error [m]
y_{26} error [m]
$\dot{x}_{25}$ [m/s]
$\dot{y}_{26}$ [m/s]
$\dot{x}_{25}$ error [m/s]
$\dot{y}_{26}$ error [m/s]
t [s]


estimated
simulated
$|X_{25}|$
$|Y_{26}|$
Frequency [Hz]

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