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Data-Driven Model Order Reduction for real-time multibody simulations

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ABSTRACT *Currently, multibody models are the state of the art to describe the dynamics of complex mechanical systems. However, due to the nonlinear characteristics and the relatively large number of Degrees Of Freedom (DOFs), multibody simulations typically cannot be run in real-time. In this work, a novel data-driven local Model Order Reduction (MOR) approach is proposed, in order to perform most of the computations offline and obtain online real-time capabilities. Using displacement data either from experiments or simulations, a local Principal Component Analysis (PCA) is performed. The obtained Reduced Order Bases (ROBs) permit to locally reduce the system to a linear model and to constrain its motion without the necessity of additional equations. Thus, the dynamics can be linearly approximated with a reduced number of DOFs in a system of Ordinary Differential Equations (ODEs) that can be solved in real-time. Application examples of the technique show its increased velocity with respect to the solution of a full multibody simulation while maintaining results accuracy.*

1 Introduction

Over the past decades numerical simulations are frequently exploited to reduce the design cost and limit the number of physical prototypes required during mechanical design. In particular, for the dynamic simulations of complex mechanical systems the state of the art is represented by multibody models. More recently, these modelling approaches have found new fields of application in model-based state-estimation and control. However, for these applications the multibody models typically require a relatively large number of Degrees Of Freedom (DOFs) in order to accurately represent the system dynamics, leading to non-real-time simulation times. This is further aggravated by the typical differential-algebraic structure of the resulting equations of motion for multibody models, which are difficult to handle in many of these estimation and control algorithms. In this work we will focus on a novel multibody model reduction approach which is more suitable for these applications.

A possible solution for the high computational load is to find a smaller number of DOFs able to represent the system with sufficient accuracy at a lower computational cost: such procedure is called Model Order Reduction (MOR), as outlined in [1]. In case of nonlinear systems, an approach is to linearize the model around particular pre-computed configurations that are then used to speed up the online simulations, as in the Trajectory Piecewise Linear Approximation (TPWL) [2] and the Global Modal Parametrization (GMP) [3, 4] methods.

In this work, a local data-driven MOR approach is proposed, based on the Principal Component Analysis (PCA) [5] and Proper Othogonal Decomposition (POD) [6]. The proposed approach exploits a pre-computed solution database to obtain an approximated prediction during the online simulation. The submanifolds for the local reduction are obtained through the clustering of the solution space in which the model is then locally reduced for each cluster. During the online simulation, a consistent transition between the different locally Reduced Order Models (ROMs) is ensured through an energy-preserving projection [7]. The application to two rigid model examples shows that the proposed approach is promising and can meet some of the requirements in model structure for control and estimation applications.

The paper is structured as follows: Section 2 illustrates the proposed MOR procedure, Section 3 shows the results of application examples and Section 4 reports some concluding remarks.

2 Local Model Order Reduction

In general, the dynamics of a multibody system model can be described as:

$$\boldsymbol{M}\,\ddot{\boldsymbol{q}} + \boldsymbol{f}_{int}(\boldsymbol{q}) + \Phi_q^{\mathrm{T}}\,\boldsymbol{\lambda} = \boldsymbol{f}_{ext} \tag{1}$$

$$\Phi(\boldsymbol{q}) = 0 \tag{2}$$

Where, given n DOFs and n_c constraints, $\boldsymbol{q} \in \mathbb{R}^n$ is the vector of generalized coordinates of the system and the double-dot accent indicates the second time derivative, $\boldsymbol{M} \in \mathbb{R}^{n\times n}$ is the mass matrix, $\boldsymbol{f}_{int} \in \mathbb{R}^n$ and $\boldsymbol{f}_{ext} \in \mathbb{R}^n$ are respectively the vectors of internal and external forces, $\Phi \in \mathbb{R}^{n_c}$ is the constraints vector and $\Phi_{\boldsymbol{q}}$ is its Jacobian with respect to $\boldsymbol{q}$, and $\boldsymbol{\lambda} \in \mathbb{R}^{n_c}$ is the vector of Lagrange multipliers.

The generalized coordinates $\boldsymbol{q}$ are used to identify the configuration of the system and different possibilities to define them, as sum of positional and angular coordinates, have been proposed in literature. In particular, some of them, mainly based on only position coordinates, allow to obtain a constant mass matrix as explained in [8, 9] for rigid systems and in [10] for flexible ones. The constant mass matrix feature is exploited in the MOR approach introduced hereafter. We therefor assume the availability of a set of representative position coordinates for each body, either from the simulation coordinates or obtained through post-processing.

While multibody models offer a widely applicable formalism, they typically cannot run in (hard) real time due to the nonlinearity, the large number of DOFs, and the differential algebraic equation structure. Thus, for the purpose of performing real-time dynamic simulations, a local MOR technique is proposed in this work. This approach can be summarized in two different phases, an offline phase where most of the computations are pre-executed and the reduced order model is *trained*; and a second online phase where the reduced order simulation is performed:

1. Offline pre-computations:
   (a) Training data generation with full order model (FOM);
   (b) Training data clustering;
   (c) Local Model Order Reduction definition.
2. Online simulation (for each time step):
   (a) Recall the Reduced Order Model related to the current simulation state;
      - If necessary, perform energy-preserving projection from one model cluster to another one.
   (b) Solve the linear dynamics equation in time.

These different steps are elaborated further in the following sections.

2.1 Offline pre-computations

2.1.1 Training data generation

Firstly, a representative set of regular multibody simulations of the system model are performed. These simulations are expressed in generalized coordinates $\boldsymbol{q}$ and will lead to a set of training vectors $\boldsymbol{Q}_{tr}$. These generalized coordinates typically represent a combination of position and angular coordinates. In this work we assume an initial transformation is made into position-only coordinates as, for example, the natural coordinates introduced in [9]. This assumption is necessary as the proposed model order reduction procedure assumes a meaningful use of the Euclidean distance. If angular DOFs are used, such Euclidean distance is arbitrary and cannot be exploited. However, if position DOFs are used, consistent results can be obtained.

Therefor, in the following sections it is supposed that $\boldsymbol{q}$ indicates position coordinates and the Eculidean distance has meaning on the training dataset $\boldsymbol{Q}_{tr}$.

2.1.2 Training data clustering and reduction

Data clustering Next, a clustering technique [11] is used in order to separate the training displacement space $\boldsymbol{Q}_{tr}$ in linear subdomains. In this case the K-Means clustering [12], using the $L2$ norm (or Euclidean distance) as similarity measure between the displacements, is applied. The clusters and associated centroids are obtained as:

$$\arg\min_{\mathscr{P}} \sum_{i=1}^{n_p} \sum_{\boldsymbol{q}\in\mathscr{P}^i} ||\boldsymbol{q}-\mu^i|| \tag{3}$$

$$\mu^i = \frac{1}{n^i}\sum_{j=1}^{n^i} \boldsymbol{q}^j,\ \forall \boldsymbol{q^j} \in \mathscr{P}^i \tag{4}$$

Where, given a training set $\boldsymbol{Q}_{tr}$ from which all $\boldsymbol{q}$ are drawn, and given the desired number of clusters n_p, $\mathscr{P}^i$ are the obtained clusters and μ^i are the corresponding centroids, as defined in Eq. (4). The initial guesses of the centroids for the optimization of Eq. (3) can be chosen following the methodology outlined by Arthur *et al.* [13]. From this clustering algorithm each cluster $\mathscr{P}^i$ has a centroid μ^i and allocated training vectors $\boldsymbol{Q}^i_{tr}$:

$$\mathscr{P}^i = \left\{\mu^i,\ \boldsymbol{Q}^i_{tr}\right\} \tag{5}$$

After the application of the clustering procedure, the reduced order basis setup can be locally performed for each cluster, as presented in the following paragraph.

Reduced order basis definition At this point, a local principal component analysis (PCA) can be performed for each cluster separately: the sets of Reduced Order Bases (ROBs) are obtained through the Singular Value Decomposition (SVD) of the training displacements $\boldsymbol{Q}^i_{tr}$ assigned to the different clusters i:

$$[\boldsymbol{U}^i, \boldsymbol{S}^i, \boldsymbol{W}^i] = \mathrm{svd}(\boldsymbol{Q}^i_{tr} - \mu^i) \tag{6}$$

$$\boldsymbol{V}^i = \left\{\boldsymbol{U}^i_j \mid j \le n_r\right\} \tag{7}$$

$$\boldsymbol{q} \approx \boldsymbol{V}^i\,\boldsymbol{q}_r + \mu^i \tag{8}$$

Where, given n^i points $\boldsymbol{q}$ belonging to the training set for a cluster i, $\boldsymbol{U} \in \mathbb{R}^{n\times n}$, $\boldsymbol{S} \in \mathbb{R}^{n\times n^i}$, $\boldsymbol{W} \in \mathbb{R}^{n^i\times n^i}$ are the result of the SVD factorisation of the displacements matrix and, given the number of reduced DOFs n_r, the ROB $\boldsymbol{V}$ is composed by the first n_r singular vectors as in Eq. (7). The relation between the reduced order coordinates $\boldsymbol{q}_r$ and the full order position coordinates $\boldsymbol{q}$ is then represented by Eq. (8).

By means of the obtained ROBs, the FOM can be locally reduced and its number of DOFs decreased. Using these ROBs, the local reduced order models are defined as discussed in the following section.

2.1.3 Local reduced order model definition

In the proposed MOR approach, we assume that locally on each cluster, the multibody dynamics behaviour can be described by an unconstrained linear model. In order to obtain these local linear models, a set of model system matrices need to be approximated for the different cluster. How this approximation is performed, is discussed in the following paragraphs.

Kinetic energy and reduced order mass matrix Under the assumption of a locally linear reduced order model in cluster i, the kinetic energy e_{kin} can be expressed for the reduced order coordinates as $\boldsymbol{q}_r$:

$$e_{kin}(\dot{\boldsymbol{q}}_r) = \frac{1}{2}\,\dot{\boldsymbol{q}}_r^{\mathrm{T}}\,M^i_r\,\dot{\boldsymbol{q}}_r \tag{9}$$

Where $\boldsymbol{M}_r^i \in \mathbb{R}^{n_r \times n_r}$ is the reduced order mass matrix which is obtained, for each cluster, from the balance of the inertia forces:

$$\boldsymbol{f}_{inertia,r}(\ddot{\boldsymbol{q}}_r) = \boldsymbol{M}_r^i \, \ddot{\boldsymbol{q}}_r \tag{10}$$

$$\boldsymbol{M}_r^i = \boldsymbol{V}^{i\mathrm{T}} \, \boldsymbol{M} \, \boldsymbol{V}^i \tag{11}$$

Where $\boldsymbol{f}_{inertia,r} \in \mathbb{R}^{n_r}$ is the vector of reduced order inertia forces. Here, we assume the availability of the constant mass matrix $\boldsymbol{M}$ expressed in the position coordinates, as for example for a natural coordinates formulation [8, 9].

Potential energy and reduced order stiffness matrix For a mechanism containing springs, the potential energy e_{pot} in the reduced order coordinates for a given cluster i is approximated as:

$$e_{pot}(\boldsymbol{q}_r) = u_0^i + \frac{1}{2} \, (\boldsymbol{q}_r + \boldsymbol{q}_{r,0}^i)^T \boldsymbol{K}_r^i (\boldsymbol{q}_r + \boldsymbol{q}_{r,0}^i) \tag{12}$$

Where $u_0^i \in \mathbb{R}$ and $\boldsymbol{q}_{r,0}^i \in \mathbb{R}^{n_r}$ represent the parameters for the translation of the parabola vertex to locally approximate the energy in a certain cluster i. $\boldsymbol{K}_r^i \in \mathbb{R}^{n_r \times n_r}$ is the local reduced order stiffness matrix that can be obtained from a Galerkin projection on the full order elastic forces:

$$\boldsymbol{f}_{el,r}(\boldsymbol{q}_r) = \boldsymbol{K}_r(\boldsymbol{q}_r + \boldsymbol{q}_{r,0}) = \boldsymbol{V}^{\mathrm{T}} \, \boldsymbol{f}_{el}(\boldsymbol{q}) \tag{13}$$

$$\Rightarrow \boldsymbol{K}_r \, \boldsymbol{q}_{r,0} = \boldsymbol{V}^{\mathrm{T}} \, \boldsymbol{f}_{el}(\mu) \tag{14}$$

Where $\boldsymbol{f}_{el} \in \mathbb{R}_n$ is the vector of elastic forces corresponding to a certain displacement $\boldsymbol{q}$ and the subscript r indicates its reduced order version. Knowing that $\boldsymbol{q}_r = 0$ implies $\boldsymbol{q} = \mu$ from Eq. (8), the substitution in Eq. (13) leads to Eq. (14). This identity allows to set up the following optimization problem for identifying the reduced order stiffness matrix, repeated for each cluster:

$$\forall \boldsymbol{q}, \; \mu^i \in \mathscr{P}^i :$$

$$\boldsymbol{q}_r = \boldsymbol{V}^{i\mathrm{T}}(\boldsymbol{q} - \mu^i) \tag{15}$$

$$\underset{\boldsymbol{K}_r^i}{\text{minimize}} \quad \|\boldsymbol{K}_r^i \boldsymbol{q}_r - (\boldsymbol{V}^{i\mathrm{T}} \boldsymbol{f}_{el}(\boldsymbol{q}) - \boldsymbol{V}^{i\mathrm{T}} \boldsymbol{f}_{el}(\mu^i))\|_2 \tag{16}$$

$$\text{subject to} \quad \boldsymbol{K}_r^i \succeq 0$$

In order guarantee a stable reduced order model, a positive semi-definiteness constraint needs to be added for the reduced order stiffness matrix. Such optimization can be executed following the procedure presented in [14].

Then, after the local $\boldsymbol{K}_r^i$ are obtained, the values of the parameters $\boldsymbol{q}_{r,0}$ and u_0 for each cluster can be found with the substitutions in Eq. (14) and Eq. (12), respectively.

Reduced order damping matrix In case that velocity-dependent forces are present in the system, a locally linear reduced order damping matrix is identified, for each cluster i:

$$\boldsymbol{f}_{vel,r}(\dot{\boldsymbol{q}}_r) = \boldsymbol{V}^{i\mathrm{T}} \, \boldsymbol{f}_{vel}(\dot{\boldsymbol{q}}) \tag{17}$$

$$\forall \dot{\boldsymbol{q}}, \; \dot{\boldsymbol{q}}_r \in \mathscr{P}^i :$$

$$\dot{\boldsymbol{q}}_r = \boldsymbol{V}^{i\mathrm{T}} \dot{\boldsymbol{q}} \tag{18}$$

$$\underset{\boldsymbol{D}_r^i}{\text{minimize}} \quad \|\boldsymbol{D}_r^i \dot{\boldsymbol{q}}_r - \boldsymbol{V}^{i\mathrm{T}} \, \boldsymbol{f}_{vel}(\dot{\boldsymbol{q}})\|_2 \tag{19}$$

$$\text{subject to} \quad \boldsymbol{D}_r^i \succeq 0$$

Where $\boldsymbol{f}_{vel} \in \mathbb{R}^n$ is the vector of velocity-dependent internal forces related to the velocity $\dot{\boldsymbol{q}}$, the subscript r indicates its reduced order version and $\boldsymbol{D}_r^i \in \mathbb{R}^{n_r \times n_r}$ are the local reduced order damping matrices. The optimization can be solved similarly to the previous section.

Reduced order external forces The reduced order external forces are computed as:

$$\boldsymbol{f}_{ext,r}(t) = \boldsymbol{V}^{i\mathrm{T}}\, \boldsymbol{f}_{ext}(t) \tag{20}$$

Where $\boldsymbol{f}_{ext} \in \mathbb{R}^n$ represents the vector of external forces at a certain time step t and the subscript r indicates its reduced order version that can be pre-computed offline for each possible cluster, or calculated online at each simulated time step.

All the operations described in this section can be pre-executed offline, thus they do not impact the online computational load. The ROM matrices are stored for all clusters and will be recalled during the online simulation described in Section 2.2.

2.2 Online phase

During the online phase, the reduced order simulation is performed. At each time step, the cluster and the linked ROM related to the current state of the simulation are recalled. For the initial time step, it is assumed that the conditions of the system are known. Thanks to the introduced MOR procedure, the model nonlinear dynamics is piecewise linearly approximated and can be described through a set of low order Ordinary Differential Equations (ODEs) that can be easily solved in real time. The complete procedure is explained in this section.

2.2.1 Local reduced order model selection and projection

At each time step t, the vector of the approximated full-order coordinates $\tilde{\boldsymbol{q}}$ can be obtained through the back-projection:

$$\tilde{\boldsymbol{q}}(t) = \boldsymbol{V}^i\, \boldsymbol{q}_r(t) + \mu^i \tag{21}$$

The initial conditions $\boldsymbol{q}(t_0)$ are supposedly known and used for the first time step. Then, the cluster and related ROM for the next time step are selected according to the centroid closest to the state of the simulation:

$$\min_{\mu^i \in \mathscr{P}} ||\tilde{\boldsymbol{q}}(t) - \mu^i|| \tag{22}$$

Then, the ROBs and reduced order mass, stiffness, damping matrices related to the corresponding cluster are re-called and the ROM is updated. If the new cluster is different from the one of the previous time step, an energy-preserving projection into the new ROM is performed according to the procedure below. Without this projection, the resulting simulation would rapidly become either highly dissipating or unstable. In addition, for the purpose of maintaining as much as possible the momentum of motion, the energy-preserving scaling is not applied to the common subspace between the new and previous ROBs since these directions are not affected by the projection.

Firstly, the common subspace $\boldsymbol{V}_c$ dimension and basis must be found:

$$n_d = \dim(\boldsymbol{V}_c) \tag{23}$$

$$\boldsymbol{V}_c = \boldsymbol{V} \cap \boldsymbol{V}_{prev} \tag{24}$$

Where $\boldsymbol{V}_c \in \mathbb{R}^{n \times n_d}$ represents the basis of the intersection subspace as indicated by Eq. (24), $\boldsymbol{V}$ is the ROB associated with the new ROM while the subscript *prev* indicates the ROB associated with the previous ROM.

According to [15], $\boldsymbol{V}_c$ can be obtained as:

$$[\boldsymbol{Y}, \boldsymbol{\Lambda}, \boldsymbol{Z}^{\mathrm{T}}] = \mathrm{svd}(\boldsymbol{V}_{prev}^{\mathrm{T}}\, \boldsymbol{V}) \tag{25}$$

$$n_d = \dim(\mathrm{diag}(\boldsymbol{\Lambda}) > 1 - \varepsilon) \tag{26}$$

$$\boldsymbol{Q} = \boldsymbol{V}_{prev}\, \boldsymbol{Y} \tag{27}$$

$$\boldsymbol{V}_c = \left\{ \boldsymbol{Q}_j \mid j \leq n_d \right\} \tag{28}$$

Where ε is a small parameter chosen as tolerance and V_c is composed by the first n_d columns of $\boldsymbol{Q}$ as indicated in Eq. 28. In order to reduce the amount of online computations, such SVD can be pre-computed offline for all the adjacent clusters.

Then, the position reduced order coordinates can be projected into the new ROM, imposing the potential energy preservation:

$$\begin{aligned} \underset{\boldsymbol{q}_r}{\text{minimize}} \quad & \|(\boldsymbol{V}\ \boldsymbol{q}_r + \mu) - (\boldsymbol{V}_{prev}\ \boldsymbol{q}_{r,prev} + \mu_{prev})\|_2 \\ \text{subject to} \quad & e_{pot}(\boldsymbol{q}_r) = e_{pot}(\boldsymbol{q}_{r,prev}) \\ & \boldsymbol{V}_c^{\mathrm{T}}\ (\boldsymbol{V}\ \boldsymbol{q}_r) = \boldsymbol{V}_c^{\mathrm{T}}\ (\boldsymbol{V}_{prev}\ \boldsymbol{q}_{r,prev}) \end{aligned} \tag{29}$$

Where $\boldsymbol{q}_{r,prev}$ represents the reduced order coordinates projected into the previous ROB $\boldsymbol{V}_{prev}$ and the potential energy e_{pot} is calculated according to Eq. (12).

A similar procedure is followed for the velocity reduced order coordinates, projected into the new ROM imposing the kinetic energy preservation:

$$\begin{aligned} \underset{\dot{\boldsymbol{q}}_r}{\text{minimize}} \quad & \|\boldsymbol{V}\ \dot{\boldsymbol{q}}_r - \boldsymbol{V}_{prev}\ \dot{\boldsymbol{q}}_{r,prev}\|_2 \\ \text{subject to} \quad & e_{kin}(\dot{\boldsymbol{q}}_r) = e_{kin}(\dot{\boldsymbol{q}}_{r,prev}) \\ & \boldsymbol{V}_c^{\mathrm{T}}\ (\boldsymbol{V}\ \dot{\boldsymbol{q}}_r) = \boldsymbol{V}_c^{\mathrm{T}}\ (\boldsymbol{V}_{prev}\ \dot{\boldsymbol{q}}_{r,prev}) \end{aligned} \tag{30}$$

Where $\dot{\boldsymbol{q}}_{r,prev}$ represents the reduced order velocities projected into the previous ROB $\boldsymbol{V}_{prev}$ and the kinetic energy e_{kin} is calculated according to Eq. (9).

Finally, the reduced order acceleration is projected into the new ROM according to:

$$\ddot{\boldsymbol{q}}_r = \boldsymbol{V}^{\mathrm{T}}(\boldsymbol{V}_{prev}\ \ddot{\boldsymbol{q}}_{r,prev}) \tag{31}$$

Where $\ddot{\boldsymbol{q}}_{r,prev}$ represents the reduced order accelerations projected into the previous ROB $\boldsymbol{V}_{prev}$.

2.2.2 Time-domain solution of local reduced order model

At this point, the reduced order equation of motion can be solved. Since the motion of the model is allowed only along the obtained principal components of the ROBs, additional constraint equations are not necessary and the ROM dynamics can be linearly approximated and described through simple ODEs:

$$\boldsymbol{M}_r\ \ddot{\boldsymbol{q}}_r + \boldsymbol{f}_{int,r} = \boldsymbol{f}_{ext,r} \tag{32}$$

$$\boldsymbol{f}_{int,r} = \boldsymbol{K}_r\ (\boldsymbol{q}_r + \boldsymbol{q}_{r,0}) + \boldsymbol{D}_r\ \dot{\boldsymbol{q}}_r \tag{33}$$

Which can be solved through explicit integration schemes, e.g. the central difference method, to enable hard real-time performance. The full resulting time-integration procedure for the proposed ROM is summarized in Algorithm 1.

3 Application Examples

In order to validate the proposed reduced order modelling methodology, it is applied to two application examples.

The first is a rigid pendulum model with a carriage constraint that restricts the motion to a translation along the x axis and a rotation in the yz plane, as shown in Fig. 1. The training is performed with a x-translational motion due to a force $\boldsymbol{f}_x$ and a yz-rotational motion due to a torque around the x-axis $\boldsymbol{t}_x$. The procedure described in Section 2 is followed. Initially, the clustering is applied to the displacement data as displayed in Fig. 2, and consequently PCA can be locally carried out.

Algorithm 1 Piece-wise time-domain integration

for $t_i = t_1 : t_{end}$ **do**
  Find local ROM(t_i), Eq. 22
  if ROM(t_i) different from ROM(t_{i-1}) **then**
    Update $\boldsymbol{M}_r$, $\boldsymbol{D}_r$, $\boldsymbol{K}_r$, $\boldsymbol{q}_{r,0}$, u_0
    Energy-preserving projection of position, Eq. 29
    Energy-preserving projection of velocity, Eq. 30
    Projection of acceleration, Eq. 31
  end if
  Integrate:

$$\Delta t = t_i - t_{i-1} \tag{34}$$

$$\boldsymbol{q}_r(t_i) = \boldsymbol{q}_r(t_{i-1}) + \Delta t\ \dot{\boldsymbol{q}}_r(t_{i-1}) + \frac{\Delta t^2}{2}\ \ddot{\boldsymbol{q}}_r(t_{i-1}) \tag{35}$$

$$\boldsymbol{f}_{int,r}(t_i) = \boldsymbol{K}_r\ (\boldsymbol{q}_r(t_i) + \boldsymbol{q}_{r,0}) + \boldsymbol{D}_r\ \dot{\boldsymbol{q}}_r(t_{i-1}) \tag{36}$$

$$\ddot{\boldsymbol{q}}_r(t_i) = \boldsymbol{M}_r^{-1}\ (\boldsymbol{f}_{ext,r}(t_i) - \boldsymbol{f}_{int,r}(t_i)) \tag{37}$$

$$\dot{\boldsymbol{q}}_r(t_i) = \dot{\boldsymbol{q}}_r(t_{i-1}) + \frac{\Delta t}{2}\ (\ddot{\boldsymbol{q}}_r(t_{i-1}) + \ddot{\boldsymbol{q}}_r(t_i)) \tag{38}$$

end for

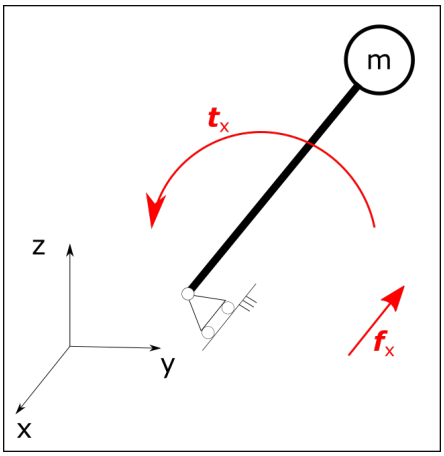

Fig. 1: The example pendulum model.

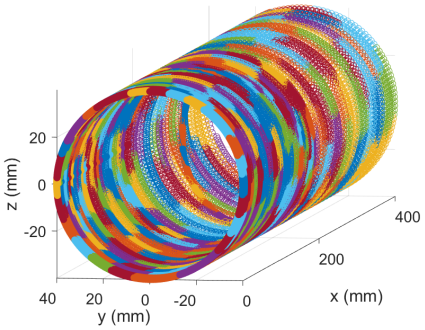

Fig. 2: Result of the clustering applied to the solution space.

Then, the reduced order simulation is performed recalling in each cluster the associated locally linear ROM, as shown in Fig. 3. Finally, in Fig. 4, the back-projected Centre Of Gravity (COG) displacement from the ROM simulation is compared with a FOM multibody simulation. The comparison shows that the proposed ROM technique, with the energy-preserving projection, gives a good approximation of the FOM generalized coordinates.

The second example is a rigid MacPherson suspension model, schematized in Fig. 5, to which a z-axis sinusoidal force acting on the wheel spindle is applied . The model has six bodies: the lower control arm is linked to the chassis by two spherical joints; the steering knuckle is linked to the lower control arm by a spherical joint, to the strut by a prismatic joint and a spring-damper element and to the tie-rod by a spherical joint. The MOR procedure is followed and the comparison of the coordinates of the knuckle COG between the ROM and FOM simulations is reported in Fig. 6. The simulation time with the proposed MOR technique is decreased, while sufficient accuracy is maintained. The methodology can be used to achieve real-time requirements.

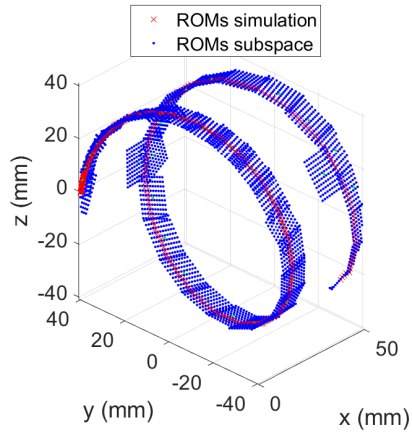

Fig. 3: Simulated displacement using the linear ROMs.

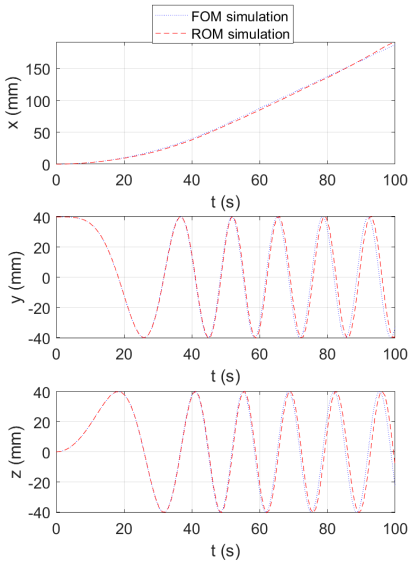

Fig. 4: Centre Of Gravity displacement, comparison between the reduced order simulation and the standard simulation.

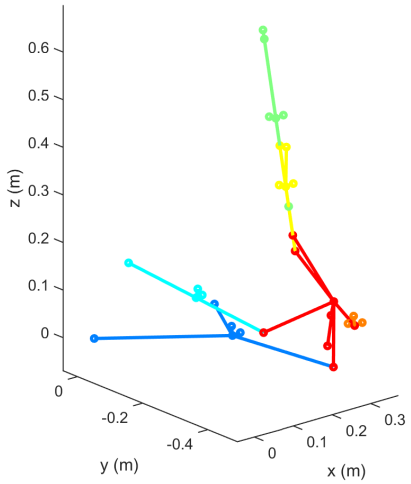

Fig. 5: Scheme of the MacPherson suspension model.

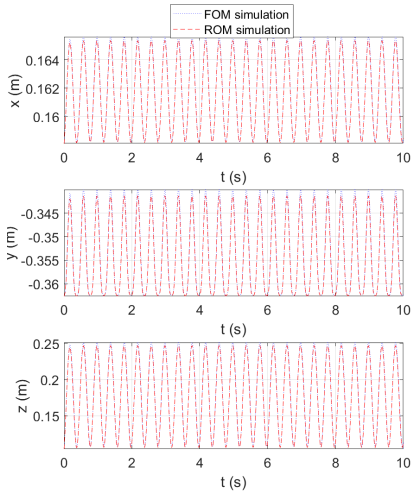

Fig. 6: Knuckle Centre Of Gravity displacement, comparison between the reduced-order simulation and the full-order simulation.

4 Conclusions

In this work, a novel data-driven model order reduction technique for real-time multibody simulations is introduced. The proposed technique is based on an offline piecewise linear reduced order model (ROM) identification and an online solution of this model. The ROM setup is based on a division of the solution space into clusters, then the system is locally reduced to a minimum amount of DOFs. During the online simulation, the ROM related to the current state is recalled and it permits to solve the dynamics equation for the reduced order DOFs at a lower computational cost. In order to allow the transfer from one locally linear ROM to another one while moving from

one cluster to another, a dedicated time integrator is setup to handle these model discontinuities.

The results presented in this work, show that a very low-cost hard real-time simulation scheme can be achieved while maintaining good accuracy of the results for two numerical examples. Further research is under development in order to increase the robustness of the technique and extend it to flexible multibody models.

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