Solution of the general single contact frictional problem in multibody dynamics using b-geometry

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Dynamics of systems possessing mechanical components that come in contact during their motion is a classical subject of Mechanics. This is due to both its large practical significance and the challenging theoretical issues arising in the effort to predict and understand the various phenomena observed and related to contact events. Previous studies have demonstrated that friction effects are responsible for the appearance and intensity of a plethora of new phenomena during contact (e.g., [1-3]).

The motion constraints related to frictional contact between mechanical components of a dynamical system are known as unilateral and are expressed by inequalities. Based on the type of approach adopted, the previous studies on the subject can roughly be divided in two general categories. In the first category, the contact phase is assumed to take place in an instantaneous manner. This leads to the appearance of a discontinuity in the velocities, accompanied by unbounded contact forces in order to avoid interpenetration. This, in turn, leads to the necessity of employing techniques of the so-called Non-smooth Mechanics (e.g., [3-6]). In essence, these approaches lead to prediction of the post-impact velocities through an algebraic process, making use of the pre-impact velocities and appropriate restitution coefficients. On the other hand, the second category of previous studies on systems with unilateral constraints is based on the Darboux-Keller approach (e.g., [2,3]). They all consider the normal impulse as an independent time-like variable and lead to an approximate set of equations of motion during the contact phase in the form of ordinary differential equations (ODEs). These methods also require the use of a restitution coefficient in order to predict the end of the contact phase [2].

In the present study, an analysis was developed leading to a solution of the dynamic problem for the general single point frictional collision between two mechanical bodies. This analysis was performed within the framework of analytical dynamics, by employing some key concepts of differential geometry [7]. First, a boundary was constructed for the original configuration manifold by using the condition of no impenetrability. Then, the essential geometric properties of the constrained manifold were determined and found to change significantly near the boundary of the configuration manifold. This provided the foundation for applying Newton's law of motion and led to an elegant geometric picture [8]. Specifically, the motion during the contact phase was found to be governed by a set of three ODEs, when expressed in a special coordinate system in the close vicinity of the configuration boundary, having one axis normal to the boundary and the remaining axes in the tangent to the boundary. The inertia of the figurative particle representing the motion of the system was found to increase rapidly as it approaches the boundary along this axis. At the same time, a strong repulsive force arises, pushing this particle away from the boundary. In addition, friction was found to activate action along two special tangential directions only, determined by the mapping with the physical space. Finally, the equations of motion in the original coordinate system were simply obtained by a proper projection of these three ODEs.

After completing the analysis, the study was focused on investigating several phenomena arising during frictional contact, by using selected examples. First, the motion of a particle hitting a plane surface was examined in a quite complete manner. Then, the emphasis was shifted on collision of rigid bodies. First, central collision of a spheroid with a rough half-space was examined. Then, conditions of eccentric collision were also considered, by analyzing motion of a rigid bar hitting a plane wall. Finally, collision of deformable bodies was also examined.

For instance, in Fig. 1a is presented the history of the normal force $\hat{f}_1$ exerted from the boundary to the figurative particle during collision. The results illustrate the effect of the boundary restoring force, represented by parameter k, for a value of the dissipation force parameter $c = 0$. The time is normalized by the total duration of the contact phase, t_f, which can be obtained in closed form [8]. The results indicate that the distribution of the normal force $\hat{f}_1$ is symmetric with respect to the line $t = t_f/2$. Also, this force reaches a plateau around the middle of the contact phase for relatively small values of k. This symmetry is broken by the presence of the boundary force dissipation parameter c, as shown in Fig. 1b, for $k = 10$. In fact, a gradual increase in the value of this parameter causes a reduction in the time interval where this force is impulsive.

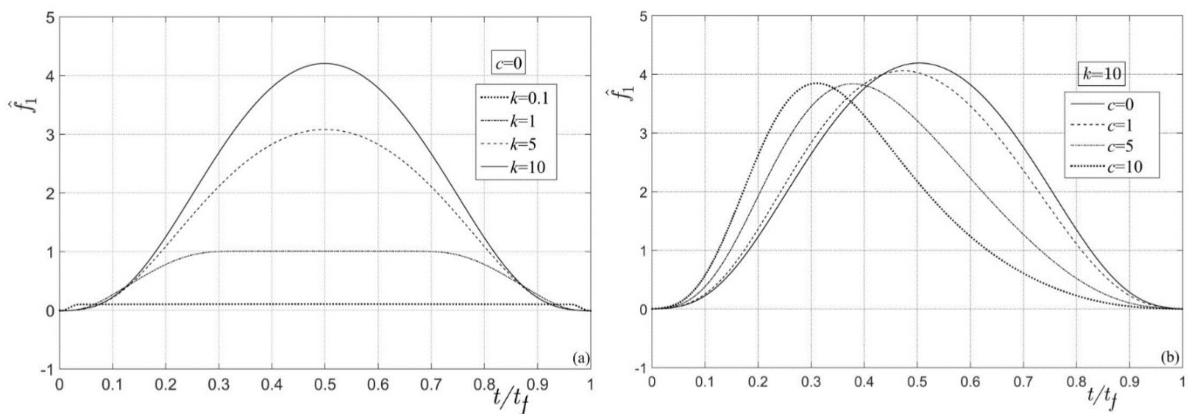

Fig. 1: History of the boundary generated normal force $\hat{f}_1$. Effect of parameter: (α) k, for $c = 0$; (b) c, for $k = 10$.

The new formulation was developed in a systematic way, which provides a firm basis for attacking the more complicated and challenging problem of multiple contacts [3-5]. In addition, the enhanced understanding provided by the geometric interpretation of the collision phenomenon studied is expected to lead to development of more accurate and efficient numerical techniques for determining the dynamics of the general class of systems examined. This is closely related to the parallel development and application of suitable contact detection methods and will also provide significant help in developing more effective and robust optimization and control algorithms.

References

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