Numerical integration of a new set of equations of motion for a class of multibody systems using an augmented Lagrangian approach

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Improving the understanding on the dynamics of constrained mechanical systems leads to useful design gains in many engineering areas. Typically, the equations of motion for this class of systems are derived and cast in the form of a set of differential-algebraic equations (DAEs) of high index. However, both the theoretical and the numerical treatment of DAEs is a delicate and difficult task [1]. The present formulation is based on a new set of equations of motion, represented by a coupled system of second order ODEs [2]. The original configuration manifold M possesses general geometric properties. Moreover, there exists a set of k motion constraints

$$\dot{\psi}^R(q,\underset{\sim}{v}) \equiv a_i^R(q)v^i = 0 \qquad (R=1,\dots,k), \tag{1}$$

where the arrays q and $\underset{\sim}{v}$ include the generalized coordinates and the components of the corresponding velocity of the system. When a constraint is holonomic, Eq. (1) can be integrated in the form $\phi^R(q)=0$. Under the action of such constraints, the equations of motion of the class of systems examined can eventually be put in the form

$$\underset{\sim}{h}_M^* = \underset{\sim}{h}_C^* \tag{2}$$

on manifold M. These quantities can be expressed in the following form over a basis of the cotangent space T_p^*M

$$\underset{\sim}{h}_M^* = [(g_{ij}v^j)^{\cdot} - \Lambda_{\ell i}^m g_{mj}v^j v^\ell - f_i]\underset{\sim}{e}^i \quad \text{and} \quad \underset{\sim}{h}_C^* = \sum_{R=1}^k a_i^R[(\bar{m}_{RR}\dot{\lambda}^R)^{\cdot} + \bar{c}_{RR}\dot{\lambda}^R + \bar{k}_{RR}\lambda^R - \bar{f}_R]\underset{\sim}{e}^i, \tag{3}$$

where the usual convention on repeated indices applies to all indices, except R [3]. The quantities λ^R are Lagrange multipliers, while g_{ij} and Λ_{ij}^ℓ represent the components of the metric and the connection on manifold M. Also, the quantities $\bar{m}_{RR}$, $\bar{c}_{RR}$, $\bar{k}_{RR}$ and $\bar{f}_R$ are specified by the action of the R-th constraint [2]. Finally, for each holonomic or non-holonomic constraint, Eq. (2) is complemented by an ODE with form

$$(\bar{m}_{RR}\dot{\phi}^R)^{\cdot} + \bar{c}_{RR}\dot{\phi}^R + \bar{k}_{RR}\phi^R = 0 \quad \text{or} \quad (\bar{m}_{RR}\dot{\psi}^R)^{\cdot} + \bar{c}_{RR}\dot{\psi}^R = 0. \tag{4}$$

For computational purposes, it is convenient to put the equations of motion (2) in the following weak form

$$\int_{t_1}^{t_2} (\underset{\sim}{h}_M^* - \underset{\sim}{h}_C^*)(\underset{\sim}{w})dt = 0, \quad \forall \underset{\sim}{w} \in T_p M. \tag{5}$$

Also, for each holonomic constraint, as expressed by Eq. (4a), the following relation is satisfied

$$\int_{t_1}^{t_2} [(\bar{m}_{RR}\dot{\phi}^R)^{\cdot} + \bar{c}_{RR}\dot{\phi}^R + \bar{k}_{RR}\phi^R]\delta\lambda^R dt = 0, \tag{6}$$

for an arbitrary multiplier $\delta\lambda^R$, while a non-holonomic constraint equation can be treated in a similar manner. Moreover, in a weak formulation, it is advantageous to consider the position, velocity and momentum variables as independent quantities [4,5]. For this, two new velocity fields, $\underset{\sim}{\upsilon}$ and $\underset{\sim}{\mu}$, are introduced, which are eventually forced to become identical to the true velocity fields $\underset{\sim}{v}$ and $\underset{\sim}{\dot{\lambda}}$. To achieve this, Eq. (5) is augmented by the terms

$$\int_{t_1}^{t_2} [\pi_i(\delta\upsilon^i - \delta v^i) + \delta\pi_i(\upsilon^i - v^i)]dt \quad \text{and} \quad \int_{t_1}^{t_2} [\sigma_R(\delta\mu^R - \delta\dot{\lambda}^R) + \delta\sigma_R(\mu^R - \dot{\lambda}^R)]dt, \tag{7}$$

where π_i and σ_R represent new sets of Lagrange multipliers, while $\delta\pi_i$ and $\delta\sigma_R$ are components of covectors belonging to the same vector space as those with components π_i and σ_R, respectively. Next, appending Eqs. (6) and (7) to Eq. (5) yields the weak form of the equations of motion for the class of systems examined as a three field set of equations for the independent position, velocity and momentum type quantities. Finally, all these lead to an augmented Lagrangian formulation [6-8], in a natural way, by just adding suitable penalty terms

$$\int_{t_1}^{t_2} \underset{\sim}{h}_P^*(\underline{w})\,dt = 0, \quad \forall \underline{w} \in T_p M \, , \tag{8}$$

where, for holonomic constraints

$$\underset{\sim}{h}_P^* = \sum\nolimits_{R=1}^{k} a_i^R \rho_R [(\bar{m}_{RR}\dot{\phi}^R)^{\cdot} + \bar{c}_{RR}\dot{\phi}^R + \bar{k}_{RR}\phi^R]\, \underset{\sim}{e}^i \, . \tag{9}$$

The weak form developed provides a firm basis for constructing appropriate numerical discretization schemes, leading to improvements over existing schemes. The validity and efficiency of such a scheme was tested and illustrated by applying it to a large number of example mechanical systems. First, it was verified that the scheme developed passes successfully all the tests related to a special set of benchmark problems, chosen by the multibody dynamics community [9]. In addition, the new scheme was also applied successfully to solve large scale industrial applications. For instance, in Fig. 1 are presented results for a complex ground vehicle model, executing a typical double lane change maneuver. These results were compared with results obtained by applying two state of the art numerical codes (i.e., ADAMS and MotionSolve). Both of these codes set up the equations of motion and solve them numerically as a system of DAEs. The vehicle model shown in Fig. 1a is composed of a basic powertrain system, a complex steering system, together with involved front and rear suspension systems with jounce and rebound bumpers. Also, the tires were modeled using the well-known Pacejka tire model. Here, some characteristic results obtained on tire forces and car velocities are presented and compared in Fig. 1b, where the results of the new method are labeled by LMD.

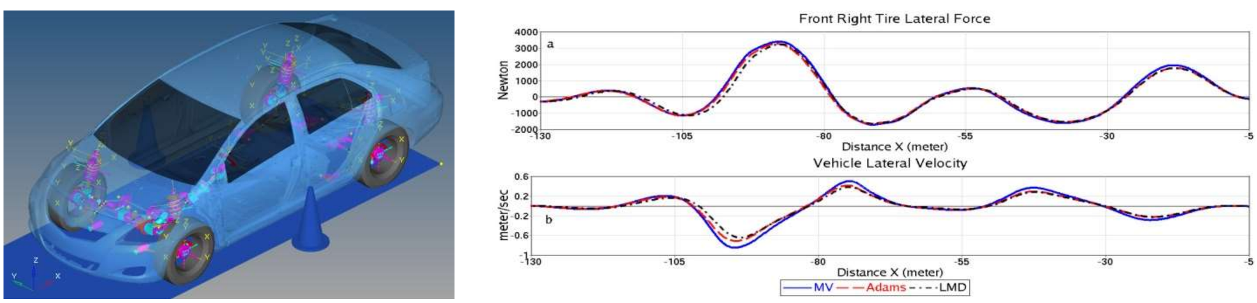

Fig. 1: Numerical results for a real car model: (a) Front right tire lateral force and (b) vehicle lateral velocity.

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