Computing Inter-Body Constraint Forces in Recursive Multibody Dynamics

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The choice of a dynamics modeling approach is a key step towards solving the equations of motion of multibody systems as required for time-domain simulations. A popular choice of a dynamics model is the non-minimal coordinates approach that uses absolute coordinates for the individual bodies, and treats the inter-body hinges as explicit bilateral constraints on the system dynamics. This approach uses Lagrange mulitpliers to compute the inter-body constraint forces as part of the solution process, but requires a DAE solver to integrate the non-minimal coordinates.

An alternative approach is to use minimal coordinate dynamics models. The recursive methods avoid the need for Lagrange multipliers, and directly solve for the generalized accelerations. This approach is made possible by the rich underlying structure of the dynamics model that allows for the analytical factorization and inversion of the mass matrix [1]. Despite the availability of faster O(N) recursive algorithms [1, 2] for solving the equations of motion, and the ability to use simpler ODE solvers, the added complexity of such models has been a deterrence to their wider use. We focus in this paper on an additional perceived barrier for these methods - the omission of the computation of inter-body constraint forces in the solution process, especially when such constraint forces are needed for monitoring internal stresses or for computing frictional forces. The often-time perception is that additional expensive computations are needed to compute these constraint forces, and that these additional costs remove the computational advantages of the recursive methods.

In this paper we address this criticism of the recursive methods and show that they are completely unfounded. We show that there are simple and very low cost methods available to compute the constraint forces should the need arise when using recursive methods. The methods directly use the articulated body algorithm quantities that are by products of the recursive solution process. The main expression for computing the inter-body constraint forces has the form

$$\mathfrak{f}(k) = \mathcal{P}(k)\mathfrak{z}(k) + \alpha(k) \tag{1}$$

Here $\mathfrak{f}(k)$ denotes the inter-body constraint spatial force between the k^{th} body and its parent, while $\mathcal{P}(k)$, $\mathfrak{z}(k)$ and $\alpha(k)$ are articulated body quantities available from the recursions used in solving the equations of motion. This expression provides a very inexpensive way to compute the constraint force - and needs only be used only when such forces are explicitly needed! While Eq. 1 is not new for tree-topology rigid body multibody systems [1, 3], it is not well known, leading to the above mentioned misconceptions about the recursive methods.

In this paper we look in further detail at the topic of computing constraint forces for multibody systems. For rigid, tree multibody systems we explain the basis for Eq. 1 and its derivation. We also derive additional useful variants of this expression.

We next examine the same topic of constraint forces for more general multibody systems. We begin by looking at the case when there are non-rigid flexible bodies in a tree-topology multibody system. We show that a form of Eq. 1 continues to apply in this case.

We next look at closed-chain systems. We pursue two paths for such systems. The first is the tree-augmented dynamics model, where the system is decomposed into a spanning tree together with additional cut-joint constraints. We derive an extension of the above approach for computing the inter-body constraint forces for such dynamics models.

An alternative dynamics modeling approach for closed-chain systems is the more recently developed constraint embedding approach. In this approach, the original graph topology is converted into a tree-topology system using body aggregation and compound bodies. The resulting minimal-coordinates dynamics model can be solved using a form of the standard recursive methods. We extend the above approach for solving for constraint forces to such constraint embedding based dynamics models.

References

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