Two-dimensional optimal motions of a two-body system

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Optimal two-dimensional motions of a system consisting of a rigid body and a point mass are considered in the absence of external forces. The system is subjected only to internal forces due to the interaction between the bodies. Optimal motions of a point mass relative to the body are obtained that correspond to the minimum time transfer of the system from a given initial state to the prescribed terminal state under the constraint imposed on the relative velocity of the point mass. Optimal controls are obtained in an explicit form. Optimal trajectories of the point mass relative to the body are circular arcs. The results obtained can be applied to spacecraft and mobile robots that can be made hermetic.

Displacement of a body in a resistive medium can be controlled by specific motions of auxiliary internal masses inside the body. This principle of locomotion is considered in a number of papers and applied to microrobots, see, for example, [1] . Optimal one-dimensional motions of bodies containing internal moving masses are analyzed in [2, 3, 4]. Two-dimensional motions of a body controlled by moving internal masses are considered in [5].

In this paper, optimal two-dimensional motions of a rigid body containing a movable mass are analyzed in the absence of external forces.

Consider a system consisting of a rigid body P of mass M and a point Q of mass m. Both bodies can move in a plane OXY. The system is at rest at the initial time moment $t = 0$. Hence, the center of mass of system $P+Q$ stays at rest and can be chosen as the origin O of the inertial Cartesian coordinate system OXY.

Denote by C the center of mass of body P and introduce the Cartesian coordinate system Cxy connected with body P. Let $\mathbf{r}$ be the radius vector of point Q relative to point C. We assume that point mass Q equipped with an actuator can move relative to body P with relative velocity $\dot{\mathbf{r}}$ bounded by constraint $|\dot{\mathbf{r}}| \leq V$.

On the strength of the conservation of momentum and angular momentum for system $P+Q$, we arrive at the equation

$$(J/M+\mu r^2)\omega+\mu\mathbf{r}\times\dot{\mathbf{r}}=0, \quad \mu=m/(M+m), \tag{1}$$

where J is the moment of inertia of body P relative to the axis passing though point C and perpendicular to plane OXY, and ω is the angular velocity of body P. Denote by x and y the coordinates of point Q with respect to the Cartesian system Cxy and by u and v the corresponding components of velocity $\dot{\mathbf{r}}$.

Equation (1) is reduced to the system

$$\dot{x}=u, \quad \dot{y}=v, \quad \dot{\varphi}=\omega=\mu(yu-xv)\left[a^2+\mu(x^2+y^2)\right]^{-1}, \tag{2}$$

where u and v are controls bounded by the constraint

$$u^2+v^2 \leq V^2, \tag{3}$$

φ is the angle of rotation of body P in plane OXY, and $a^2 = J/M$.

Two kinds of time-optimal control problems are considered for system (2) under constraint (3). In both Problems 1 and 2, the initial data

$$x(0)=x^0, \quad y(0)=y^0, \quad \varphi(0)=\varphi^0,$$

are fixed. In Problem 1, the terminal data

$$x(T)=x^1, \quad y(T)=y^1, \quad \varphi(T)=\varphi^1,$$

are also fixed. In Problem 2, the terminal angle $\varphi(T)$ is fixed as in Problem 1, whereas the terminal values $x(T)$ and $y(T)$ are free. In both Problems, time of process T is to be minimized.

Analysis of Problems 1 and 2 is performed by means of the maximum principle [6].

For the case $\mu \ll 1$, explicit solutions of Problems 1 and 2 are obtained. The optimal trajectories of point Q relative to coordinate system Cxy are proved to be circular arcs. Optimal controls and minimum time T are found.

For the general case of arbitrary $\mu < 1$, suboptimal controls are proposed that satisfy all boundary conditions.

The optimal control problems considered above can be generalized for the cases of external forces acting upon the system.

Three-dimensional problems of control for system $P+Q$ are discussed.

The solutions obtained can be applied to mobile robotic systems controlled by internal movable masses. Note that such mobile robots can be made hermetic and used in hazardous and vulnerable environment.

The results obtained can also be useful for spacecraft controlled by movable internal masses.

References

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