Selection of generalized component modes for modally reduced flexible multibody systems

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Increasing industrial demands on reliability and efficiency of modern moving engineering devices require advanced modelling techniques during the design process. Virtually all such systems are assemblies made out of multiple components that interact with each other during operation. The forces required to execute desired motions are associated with stresses, noise and vibrations. Thus, it is insufficient to model multibody (MB) systems as rigid bodies and extract boundary forces to perform subsequent standard finite element (FE) analyses. Flexible MB simulations where the system is spatially discretized are, therefore, inevitable. However, most FE models of real-world problems contain a huge number of degrees of freedom (DOFs) that cannot be efficiently simulated without model reduction techniques.

Generalized component mode synthesis (GCMS) [1] is a promising efficient alternative to existing flexible MB formulations, such as the floating frame of reference formulation (FFRF), since it preserves a linear relationship between the displacement field and the DOFs, yielding constant system matrices, which is not the case for the FFRF implemented in most commercial flexible MB simulation packages. Moreover, it is a generic formalism, easily applicable to any MB system subjected to large reference motion but small deformations, which is the case for the majority of engineering systems, such as vehicles, robots and aircraft, since large deformations severely impair or even destruct the system and are, therefore, usually unwanted.

Both, the FFRF as well as the GCMS approximate the flexible deformation by a linear combination of component modes. In the case of the FFRF, the component modes are usually the eigenmodes of vibration limited to the frequency range of interest. If the deformation is approximated by vibration modes, the reduction matrix containing column-wise the eigenmodes is well-conditioned, since the eigenvectors are linearly independent even for repeated eigenvalues. Hence, the reduction matrix of the FFRF does not introduce numerical errors. Whereas, the GCMS reduction matrix $\boldsymbol{\Phi}$ is in many cases ill-conditioned, f. e., in the order of 10^6 to 10^{17} for the analysed beam-like models, due to linearly dependent GCMS modes that may arise due to the special structure of $\boldsymbol{\Phi}$. These linear dependencies may lead to unsolvable problems, since they preclude the factorization of the system Jacobian impossible.

There is past work concerned with a rigorous mathematical derivation of the GCMS modes and equations of motion [1, 2]. Also, the formulation has been successfully applied to engineering problems [3, 4], but the issue of linear dependencies has not received much attention despite its importance. It was marginally reported in [1, 2], but has not been addressed in the available literature. Moreover, it has been believed that dependent modes only arise for symmetric problems, which is, in general not true. Hence, the aim of this contribution is to shed light on this problem inherently present in the GCMS and give suggestions how to handle the linear dependencies appropriately.

The GCMS formulation exploits a modal superposition reduction method, where the flexible deformation is approximated by a linear combination of vibration modes to reduce the system size from a large number of DOFs to a significantly smaller one. The so called generalized component modes, or GCMS shape vectors, account not only for large rigid body motion, but also represent the deformation modes in any possible orientation, leading to a linear configuration space. The reduced set of GCMS coordinates $\boldsymbol{q}$ is related to the full set of FE nodal coordinates $\boldsymbol{c}$ via, $\boldsymbol{c} \approx \boldsymbol{\Phi}\boldsymbol{q}$ with $\dim(\boldsymbol{q}) \ll \dim(\boldsymbol{c})$. The GCMS reduction matrix $\boldsymbol{\Phi}$ contains the translational $\boldsymbol{\Phi}_\mathrm{t} \in \mathbb{R}^{3N_\mathrm{n}\times 3}$, the rotational $\boldsymbol{\Phi}_\mathrm{r} \in \mathbb{R}^{3N_\mathrm{n}\times 9}$ and the flexible $\boldsymbol{\Phi}_\mathrm{f} \in \mathbb{R}^{3N_\mathrm{n}\times 9N_\mathrm{m}}$ GCMS shape vectors, i.e. $\boldsymbol{\Phi} = [\boldsymbol{\Phi}_\mathrm{t} \quad \boldsymbol{\Phi}_\mathrm{r} \quad \boldsymbol{\Phi}_\mathrm{f}]$, where

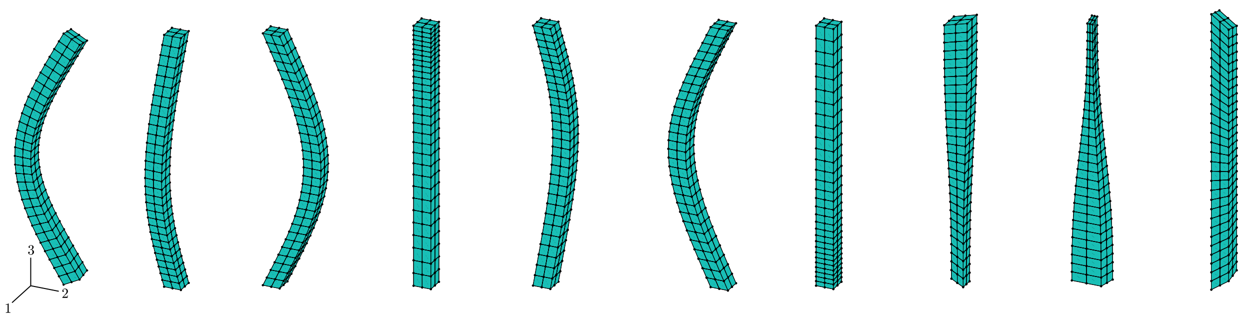
$$ {}^{\mathrm{t}}\boldsymbol{\phi}^{l} = \begin{bmatrix} \boldsymbol{e}_l^\mathrm{T} & \boldsymbol{e}_l^\mathrm{T} & \dots & \boldsymbol{e}_l^\mathrm{T} \end{bmatrix}^\mathrm{T}, \tag{1} $$

$$ {}^{\mathrm{r}}\boldsymbol{\phi}^{kl} = \begin{bmatrix} X_k^1 \boldsymbol{e}_l^\mathrm{T} & X_k^2 \boldsymbol{e}_l^\mathrm{T} & \dots & X_k^{N_\mathrm{n}} \boldsymbol{e}_l^\mathrm{T} \end{bmatrix}^\mathrm{T}, \tag{2} $$

$$ {}^{\mathrm{f}}\boldsymbol{\phi}_m^{kl} = \begin{bmatrix} \phi_m^k \boldsymbol{e}_l^\mathrm{T} & \phi_m^{k+3} \boldsymbol{e}_l^\mathrm{T} & \dots & \phi_m^{k+3(N_\mathrm{n}-1)} \boldsymbol{e}_l^\mathrm{T} \end{bmatrix}^\mathrm{T}, \tag{3} $$

with $k,l = 1,2,3$ and $m = 1,\dots,N_\mathrm{m}$ represent the columns of $\boldsymbol{\Phi}_\mathrm{t}$, $\boldsymbol{\Phi}_\mathrm{r}$ and $\boldsymbol{\Phi}_\mathrm{f}$, respectively. In Eqs. (1) to (3), $\boldsymbol{e}_l \in \mathbb{R}^3$ denotes the orthonormal set of Cartesian base vectors, X_k^i the i-th nodal coordinate of the k-th coordinate direction of the undeformed FE model, ϕ_m^k denotes the k-th component of the m-th eigenmode of vibration and N_n as well as N_m the number of nodes and eigenmodes, respectively.

The significance of the careful mode selection is illustrated with the following example. The first natural bending mode of an unconstrained square-sectioned beam and its nine corresponding flexible GCMS modes are visualized in Fig. 1. Considering the first and the second natural bending mode, which form a repeated mode pair, the first three GCMS modes of the first and second bending eigenmodes are identical. Moreover, the fourth to sixth GCMS modes are related by the factor of negative one and are therefore directly proportional. Likewise, as may be seen in Fig. 1, the set of the first three GCMS modes within each eigenmode is directly proportional to the set of the second three GCMS modes, i.e. ${}^{\mathrm{f}}\boldsymbol{\phi}_m^{1l} \propto {}^{\mathrm{f}}\boldsymbol{\phi}_m^{2l}$ with $m = 1,2$ and $l = 1,2,3$. Consequently, only nine out of the full set of 18 GCMS modes are, in this case, linearly independent.



(a) $\boldsymbol{\phi}_1$ (b) ${}^{\mathrm{f}}\boldsymbol{\phi}_1^{11}$ (c) ${}^{\mathrm{f}}\boldsymbol{\phi}_1^{12}$ (d) ${}^{\mathrm{f}}\boldsymbol{\phi}_1^{13}$ (e) ${}^{\mathrm{f}}\boldsymbol{\phi}_1^{21}$ (f) ${}^{\mathrm{f}}\boldsymbol{\phi}_1^{22}$ (g) ${}^{\mathrm{f}}\boldsymbol{\phi}_1^{23}$ (h) ${}^{\mathrm{f}}\boldsymbol{\phi}_1^{31}$ (i) ${}^{\mathrm{f}}\boldsymbol{\phi}_1^{32}$ (j) ${}^{\mathrm{f}}\boldsymbol{\phi}_1^{33}$

Fig. 1: First bending eigenmode $\boldsymbol{\phi}_1$ of an unconstrained square-sectioned beam and the nine corresponding flexible GCMS modes ${}^{\mathrm{f}}\boldsymbol{\phi}_1^{kl}$ according to Eq. (3).

It is shown in the paper, how the Cosine similarity and the Singular Value Decomposition may be used to identify and eliminate directly proportional GCMS modes, as shown in Fig. 1, and linear combinations between flexible and rigid body motion shape vectors, which may also arise.

The above example shows the importance of the mode selection process. The systematic investigation of GCMS modes, which has not been addressed in the open literature, is not only required to obtain a solvable system of equations, but enables a further reduction of the GCMS coordinates and therefore a gain in efficiency. In conclusion, the significance of GCMS modes for different problems is covered in a rigorous manner; the understanding of the mode selection shall improve the formulation's applicability. The new findings are illustrated by numerical experiments of beam-like models with different cross-sections and a crankshaft of a reciprocal combustion engine.

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