Analysis of Servo-constraints Solution Approaches for Underactuated Multibody Systems

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Trajectory tracking of multibody systems is usually pursued in terms of a two degree of freedom control structure. Ideally, the feedforward path is designed as an inverse model. It provides system inputs that exactly maneuver the system output on a specified trajectory if there were no disturbances or modeling errors. The feedforward path is appended by a state feedback path to account for possible disturbances and stabilize the motion around the specified trajectory, see Fig. 1. In case of an accurate inverse model, the tracking errors are usually small and a simple state feedback strategy such as LQR is applicable.

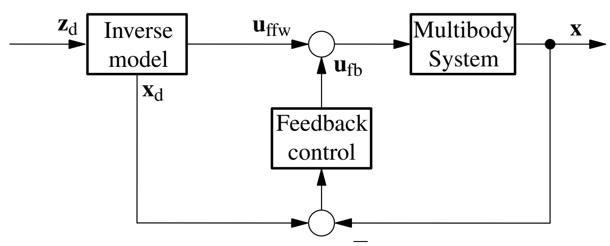


Fig. 1: Two degree of freedom control structure.

Underactuated multibody systems possess more degrees of freedom than independent control inputs. They are modeled using either minimal or redundant position and velocity coordinates $\mathbf{y}$ and $\mathbf{v}$ respectively. Due to underactuation, it is not straightforward to derive an inverse model for these systems. For example, the Byrnes-Isidori normal form approach can be applied. However, even for small and simple systems, the equations tend to become complex and difficult to analyze. Here, the servo-constraints approach is applied to derive the inverse model numerically [1]. The servo-constraints $\mathbf{c}(\mathbf{y})$ enforce the output $\mathbf{z}(\mathbf{y})$ to follow a specified trajectory $\mathbf{z}_\mathrm{d}(t)$. They append the model dynamics with geometric constraints $\mathbf{c}_\mathrm{g}(\mathbf{y})$ to form a set of differential-algebraic equations (DAEs)

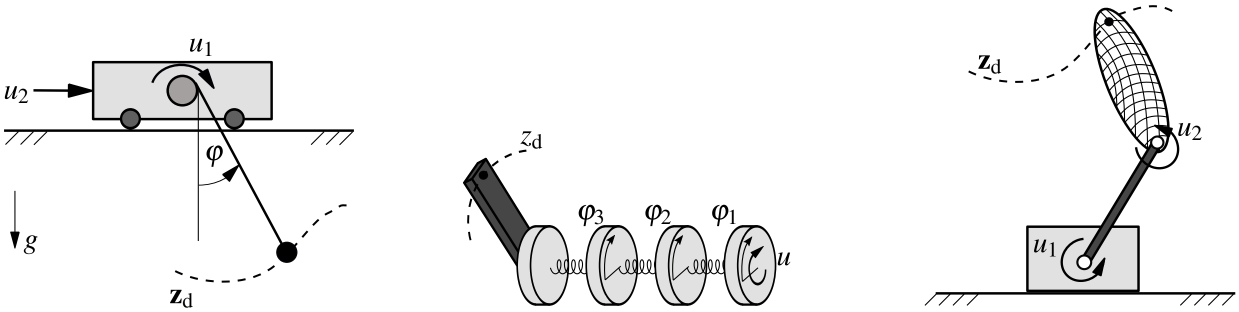
$$\dot{\mathbf{y}} = \mathbf{Z}(\mathbf{y})\,\mathbf{v} \tag{1}$$

$$\mathbf{M}(\mathbf{y},t)\,\dot{\mathbf{v}} = \mathbf{q}(\mathbf{y},\mathbf{v},t) + \mathbf{C}_\mathrm{g}^\mathrm{T}(\mathbf{y})\,\boldsymbol{\lambda} + \mathbf{B}\,\mathbf{u} \tag{2}$$

$$\mathbf{c}_\mathrm{g}(\mathbf{y}) = \mathbf{0} \tag{3}$$

$$\mathbf{c}(\mathbf{y}) = \mathbf{z}(\mathbf{y}) - \mathbf{z}_\mathrm{d}(t) = \mathbf{0}\,, \tag{4}$$

where $\mathbf{Z}$ is the kinematics matrix, $\mathbf{M}$ denotes the mass matrix, $\mathbf{q}$ describes the forces acting on the system, $\mathbf{B}$ is the input distribution matrix and $\boldsymbol{\lambda}$ denotes the Lagrangian multipliers distributed by $\mathbf{C}_\mathrm{g} = \frac{\partial \mathbf{c}_\mathrm{g}}{\partial \mathbf{y}}$. The DAEs have a comparable structure to the DAEs which are obtained from modeling multibody systems in redundant coordinates. While the geometric constraints $\mathbf{c}_\mathrm{g}(\mathbf{y})$ are enforced by the generalized reaction forces $\boldsymbol{\lambda}$, servo-constraints $\mathbf{c}(\mathbf{y})$ are enforced by the system input $\mathbf{u}$. In contrast to the reaction forces, the system input is not necessarily orthogonal to the tangent of the respective constraint manifold [1]. Thus, the arising differentiation index of the inverse model differential-algebraic Eqs. (1)-(4) might be larger than 3. Also, the dynamics of the inverse model might be much more complex than the forward dynamics.



(a) Differentially flat crane. (b) Mass-spring chain. (c) Robot with flexible body.

Fig. 2: Examples of underactuated multibody systems with either none, stable or unstable internal dynamics.

In this contribution, various arising servo-constraints solution approaches are analyzed. The solutions can be classified into different configurations. In case of differentially flat systems, the inverse model is purely algebraic. Examples for this class are cranes or undamped mass-spring chains, see Fig. 2a and Fig. 2b. Otherwise, there remains internal dynamics which is not observable from the input-output relationship. In order to apply the inverse model as feedforward control, stability of the internal dynamics has to be analyzed. The approach taken here is adopted from nonlinear control theory and is based on a coordinate transformation. Then, stability of the internal dynamics is analyzed in terms of zero dynamics. A damped mass-spring-damper chain is analyzed as an example for this system type. In order to use the inverse model in the feedforward control loop, the high index DAEs of Eqs. (1)-(4) must be solved by forward-time integration in real-time to yield the desired system inputs. This allows the specified trajectories to be changed on-line. This is possible in case of stable internal dynamics or differentially flat systems [2]. Due to large differentiation indices, there are various methods to reduce the index, such as a projection method or minimal extension. Suitable solvers are selected and analyzed in terms of real-time capability and accuracy.

In case of unstable internal dynamics, the inverse model problem yields unbounded system inputs and the approach cannot be applied directly. There are different strategies to treat unstable internal dynamics. For example, the output can be redefined to yield a slightly modified system with stable internal dynamics. On the other hand, the inverse model problem can be viewed as a non-causal system with a pre- and postactuation phase before and after the specified trajectory. This problem formulation can be solved as a boundary value or optimization problem beforehand. The similarities and differences of these two approaches are discussed and some current limitations of these approaches are analyzed. A robot arm with a flexible body visualized in Fig. 2c is analyzed as an example for this system class.

References

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[2] S. Otto and R. Seifried, “Real-time trajectory control of an overhead crane using servo-constraints,” *Multibody System Dynamics*, pp. 1–17, 2017.