Singularity-free non-redundant time integration of multibody systems models in absolute coordinate formulation

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A multibody system (MBS) is set of bodies that are subjected to constraints or restrains restricting their relative motions. Ideal bilateral geometric constraints are due to (ideal) joints or contacts. They can thus be regarded as to allow for a certain relative motion, which can be parameterized by appropriate coordinates, or as to impose certain kinematic constraints. Correspondingly, there are two different approaches to describe the MBS motion, namely the relative coordinate formulation and the absolute coordinate formulation. In the first approach, the MBS configuration is described in terms of coordinates describing the relative motions of interconnected bodies (e.g. joint variables), whereas in the second formulation the coordinates describe the 'absolute' posture of the individual bodies relative to the inertial frame. The latter leads immediately to the problem of describing spatial rigid body motions in a singularity free and at the same time computationally efficient way. This has been a long standing issue, that is traditionally addressed by using Euler-parameters (unit quaternions) [1] or more recently using natural coordinates [2]. Both parameterizations are redundant as they involve more than six variables that are consequently subjected to certain constraints.

During time integration of the equations of motions (EOM) in absolute coordinate formulations, the postures of the bodies of the MBS are updated according to their incremental motions during a time step. The incremental rotations are parameterized by the 'incremental rotation vector'. This concept was originally introduced for describing the kinematics of discretized flexible bodies and was then used within time integration schemes for spatial MBS []. The rotation vector represents canonical coordinates for spatial rotations, and the mathematical framework is the theory of Lie groups.

In time integration schemes, the kinematic equation [3]

$$\boldsymbol{\omega}^{\mathrm{b}} = \mathbf{A}(\mathbf{x})\dot{\mathbf{x}} \tag{1}$$

is solved, where $\boldsymbol{\omega}^{\mathrm{b}}(t)$ is the (body-fixed) angular velocity vector of a body and $\mathbf{x}(t)$ is the rotation vector. Now observe that the matrix $\mathbf{A}(\mathbf{x}) = \mathbf{dexp}_{-\mathbf{x}}$ is the right-trivialised differential of the exp mapping on $SO(3)$. The latter is merely the Euler-Rodriguez formula for the rotation matrix.

Solving the kinematic reconstruction equation (1) on the Lie algebra $so(3)$ thus yields the rotation the body performing when moving with angular velocity $\boldsymbol{\omega}^{\mathrm{b}}$. The important fact is that the Lie algebra of the rotation group $SO(3)$ and of the group of unit quaternions $Sp(1)$ are isomorphic. Consequently, the incremental rotation vector obtained from (1) can just as well be used to generate the Euler-parameter via the exp mapping on $Sp(1)$ describing the rotation. This was used in [4] to derived a singularity-free time stepping scheme for angular dynamics. This does allow for time integration of the EOM of an MBS under the assumption that rotations and translations are decoupled. This assumption is generally not satisfied (although rotations and translations are independent).

Rigid body motions form the Lie group $SE(3) = SO(3) \ltimes \mathbb{R}^3$. Its parameterization i terms of canonical coordinates leads to the same singularities as encountered in case of rotations. Dual quaternions, forming the Lie group $\widehat{Sp}(1)$, are the dual extension of ordinary quaternions. They allow for a singularity free parameterization of rigid body motions on the expense of using redundant coordinates for which they are becoming increasingly used in MBS modeling. This, however, leads to similar numerical burden for the numerical time integration.

Rigid body motions $SE(3)$ as well as unit dual quaternions $\widehat{Sp}(1)$ are parameterized by screw coordinates forming the Lie algebra $se(3)$. Since the latter is isomorphic to the Lie algebra of $\widehat{Sp}(1)$, a rigid body motion as well as a dual unit quaternion can be generated from given screw coordinates. Given the twist $\mathbf{V}^{\mathrm{b}} = (\boldsymbol{\omega}^{\mathrm{b}}, \mathbf{v}^{\mathrm{b}}) \in se(3)$ of a rigid body of the MBS, the corresponding instantaneous screw coordinates are determined by the solution of the kinematic reconstruction equations

$$\mathbf{V}^{\mathrm{b}} = \mathbf{dexp}_{-\mathbf{X}} \dot{\mathbf{X}} \tag{2}$$

where now $\mathbf{dexp}_{-\mathbf{X}}$ is the right-trivialised differential of the exp mapping [3].

In this paper a non-redundant and singularity-free integration of MBS models described in terms of non-redundant screw coordinates as well as redundant dual quaternions is presented exploiting the isomorphism of the screw algebra and the algebra of dual quaternions. Numerical simulations show are presented that confirm the performance of the method.

Acknowledgement

The first author acknowledges that this work has been partially supported by the Austrian COMET-K2 program of the Linz Center of Mechatronics (LCM). The second and third author acknowledge that this work has been partially supported by Croatian Science Foundation under the project IP-2016-06-6696

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