Sensitivity analysis of a full vehicle model using ALI3-P and Matrix-R formulations

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Sensitivity analysis of the dynamics of multibody systems is essential for design optimization and optimal control. Dynamic sensitivities, when needed, are often calculated by means of finite differences but, depending on the number of parameters involved, this procedure can be very demanding in terms of time, and the accuracy obtained can be very poor in mots cases. In this work, two fully analytical sensitivity formulations intended for industrial problems will be presented and compared in terms of accuracy and efficiency applying them to the sensitivity analysis of a full vehicle model.

Let us consider the equations of motion (EOM) depending on the vector of parameters $\boldsymbol{\rho} \in \mathbb{R}^p$. The objective function is defined in terms of the parameters and on the states $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}} \in \mathbb{R}^n$,

$$\psi = w(\mathbf{q}_F, \dot{\mathbf{q}}_F, \ddot{\mathbf{q}}_F, \boldsymbol{\rho}_F) + \int_{t_0}^{t_F} g(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \boldsymbol{\rho})\,\mathrm{dt}. \tag{1}$$

The problem is to obtain the sensitivity of such a cost function, expressed by the following gradient,

$$\nabla_{\boldsymbol{\rho}} \psi^{\mathrm{T}} = (w_{\mathbf{q}}\mathbf{q}_{\boldsymbol{\rho}} + w_{\dot{\mathbf{q}}}\dot{\mathbf{q}}_{\boldsymbol{\rho}} + w_{\ddot{\mathbf{q}}}\ddot{\mathbf{q}}_{\boldsymbol{\rho}} + w_{\boldsymbol{\rho}})_F + \int_{t_0}^{t_F} (g_{\mathbf{q}}\mathbf{q}_{\boldsymbol{\rho}} + g_{\dot{\mathbf{q}}}\dot{\mathbf{q}}_{\boldsymbol{\rho}} + g_{\ddot{\mathbf{q}}}\ddot{\mathbf{q}}_{\boldsymbol{\rho}} + g_{\boldsymbol{\rho}})\,\mathrm{dt}. \tag{2}$$

In equation (2) the derivatives of functions w and g are known, since the objective function has a known expression. On the contrary, the magnitudes $\mathbf{q}_{\boldsymbol{\rho}}$, $\dot{\mathbf{q}}_{\boldsymbol{\rho}}$, $\ddot{\mathbf{q}}_{\boldsymbol{\rho}}$ are the sensitivity matrices solution of a set of p DAE systems, called the Tangent Linear Model (TLM) of the equations of motion.

It was already mentioned before that two formulations are going to be used in this work. The first formulation of interest is the Matrix R formulation (see [1]), which writes an ODE system of equations of motion in the degrees of freedom of the system (independent coordinates). The second one will be the ALI3-P formulation (index-3 augmented Lagrangian formulation with velocity and acceleration projections), derived for holonomic and nonholonomic constraints in [2], which is an efficient and robust method to carry out the forward dynamics simulation of multibody systems modeled in dependent coordinates. It was extensively used for the real-time simulation of different systems with human and hardware in the loop, some of them including complex phenomena like flexibility [3] or contact with friction [4, 5].

In previous works, the sensitivity equations, needed to calculate (2), for these two formulations were derived, constituting fully analytical sensitivity formulations: in [6] both the forward and the adjoint sensitivity equiations for the Matrix R formulation were derived; in [7], the forward sensitivity equations of the ALI3-P formulation were derived.

Since the sensitivity equations of the ALI3-P formulation were recently derived and checked for academic systems only, the test case considered in this work is the K-LIM-08 full vehicle, modeled in MBSLIM [8] and shown in Figure 1. In the test maneuver, the vehicle travels sraight at a constant speed and the road is flat, but a transverse step is located at a certain distance of the departing point, therefore the front wheels first and the rear ones after, drop abruptly on the step. The objective function will be defined related to the fourth power vibration dose value, which is a measure of the riding comfort according to ISO 2631-1:

$$\psi = \int_{t_0}^{t_F} \dddot{z}^4 \mathrm{dt} \tag{3}$$

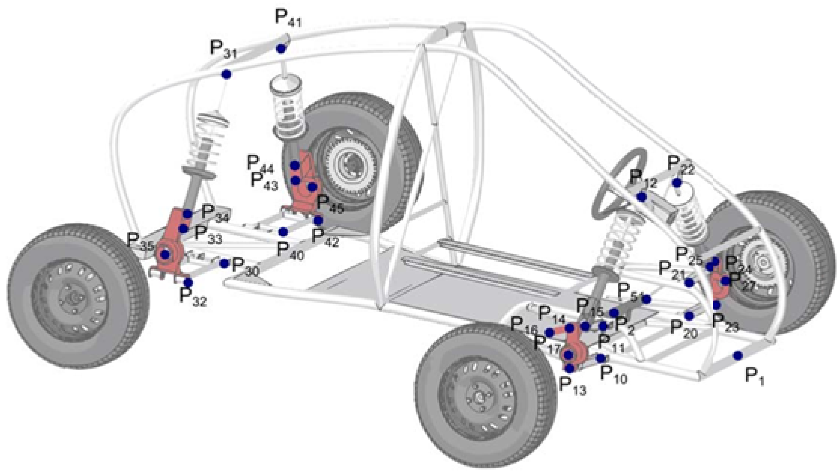

Fig. 1: Full vehicle model

where $\ddot{z}$ is the vertical acceleration of a passenger's approximate location on the chassis and t_0, t_F are the starting and final time to compute the objective funcion. As parameters to obtain the sensitivities, the natural lengths, the stiffness and the damping constants of the suspensions were chosen $\boldsymbol{\rho}^{\mathrm{T}} = \left[s_0^{front}, s_0^{rear}, k^{front}, k^{rear}, c^{front}, c^{rear}\right]$ for linear force characteristics.

The comparison of the results obtained for both formulations tests the correctness and efficiency of the novel ALI3-P sensitivity formulation with respect to the more mature Matrix R one in order to enable the sensitivity feature for all that models currently running the forward dynamics in MBSLIM with the ALI3-P formulation.

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