Thermomechanical Analysis of Interconnected Multibody Systems Using Floating Frame of Reference Formulation

Hiroki Yamashita[1], Rohit Arora[2], Hiroyuki Kanazawa[2], and Hiroyuki Sugiyama[1]

[1] *Department of Mechanical & Industrial Engineering, University of Iowa, {hiroki-yamashita, hiroyuki-sugiyama}@uiowa.edu*
[2] *Research & Innovation Center, Mitsubishi Heavy Industries, Ltd,* {rohit_arora, *hiroyuki_kanazawa*}@mhi.co.jp

Accurate prediction of the transient thermomechanical coupling behavior of deformable bodies is critically important for mechanical systems subjected to sever thermal loads. Due to high temperature that materials are exposed to, thermal deflection can cause excessive stresses and/or unintended interference between mechanical parts. For this reason, various thermomechanical coupling models have been proposed using finite element methods in the context of multi-physics simulations. For mechanical systems undergoing large rotational motion, however, nonlinear inertia effect plays an important role in design and performance evaluations, and use of fully nonlinear finite element thermomechanical coupling models requires extensive computational resources for the nonlinear time-domain dynamic analysis. For this reason, a reduced order thermomechanical model based on the Craig-Bampton component mode synthesis method [1] is extended to the floating frame of reference formulation in this study to enable the thermomechanical analysis of flexible multibody systems.

To this end, coupled structural and thermal equations of finite element models are partitioned into the internal and interface coordinates first, each of which consists of the structural and thermal coordinates. Both deformation and thermal coordinates in the internal region are then defined by a linear combination of the thermomechanical fixed-interface normal and constraint modes. Due to the thermomechanical coupling, the nodal deformation is defined as a function of the reduced-order structural and thermal coordinates, while the nodal temperature is defined by the reduced-order thermal coordinates only as follows:

$$\mathbf{q}_f = \mathbf{B}_{ff}\mathbf{p}_f + \mathbf{B}_{fT}\mathbf{p}_T \text{ and } \mathbf{q}_T = \mathbf{B}_{TT}\mathbf{p}_T \tag{1}$$

where $\mathbf{q}_f = [(\mathbf{q}_f^i)^T \quad (\mathbf{q}_f^j)^T]^T$ and $\mathbf{q}_T = [(\mathbf{q}_T^i)^T \quad (\mathbf{q}_T^j)^T]^T$ are, respectively, the physical structural and thermal coordinate vectors partitioned into the internal (i) and interface (j) regions. The vector $\mathbf{p}_f = [(\mathbf{p}_f^k)^T \quad (\mathbf{p}_f^c)^T]^T$ and $\mathbf{p}_T = [(\mathbf{p}_T^k)^T \quad (\mathbf{p}_T^c)^T]^T$ are, respectively, the reduced-order structural and thermal coordinate vectors, each of which consists of the truncated modal (k) and interface (c) coordinates. The associated projection matrices, $\mathbf{B}_{ff}$, $\mathbf{B}_{fT}$, and $\mathbf{B}_{TT}$, are defined as [2]

$$\mathbf{B}_{ff} = \begin{bmatrix} \boldsymbol{\Phi}_{ff}^{ik} & \boldsymbol{\Psi}_{ff}^{ic} \\ \mathbf{0} & \mathbf{I} \end{bmatrix},\ \mathbf{B}_{fT} = \begin{bmatrix} \boldsymbol{\Phi}_{fT}^{ik} & \boldsymbol{\Psi}_{fT}^{ic} \\ \mathbf{0} & \mathbf{0} \end{bmatrix},\ \mathbf{B}_{TT} = \begin{bmatrix} \boldsymbol{\Phi}_{TT}^{ik} & \boldsymbol{\Psi}_{TT}^{ic} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{2}$$

Using the variational principle of thermomechanical systems, equations of a reduced-order thermomechanical model using the floating frame of reference formulation, as shown in Fig. 1, can be obtained in terms of the body reference coordinates ($\mathbf{R}$ and $\boldsymbol{\theta}$) and the reduced-order structural and thermal coordinates ($\mathbf{p}_f$ and $\mathbf{p}_T$).

$$\begin{bmatrix} \mathbf{M}_{RR} & \mathbf{M}_{R\theta} & \mathbf{M}_{Rf} & \mathbf{0} \\ & \mathbf{M}_{\theta\theta} & \mathbf{M}_{\theta f} & \mathbf{0} \\ & & \bar{\mathbf{M}}_{ff} & \bar{\mathbf{M}}_{fT} \\ \text{Sym.} & & \bar{\mathbf{M}}_{Tf} & \bar{\mathbf{M}}_{TT} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{R}} \\ \ddot{\boldsymbol{\theta}} \\ \ddot{\mathbf{p}}_f \\ \ddot{\mathbf{p}}_T \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & & \bar{\mathbf{D}}_{ff} & \bar{\mathbf{D}}_{fT} \\ & \text{Sym.} & \bar{\mathbf{D}}_{Tf} & \bar{\mathbf{D}}_{TT} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{R}} \\ \dot{\boldsymbol{\theta}} \\ \dot{\mathbf{p}}_f \\ \dot{\mathbf{p}}_T \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & & \bar{\mathbf{K}}_{ff} & \bar{\mathbf{K}}_{fT} \\ & \text{Sym.} & \bar{\mathbf{K}}_{Tf} & \bar{\mathbf{K}}_{TT} \end{bmatrix} \begin{bmatrix} \mathbf{R} \\ \boldsymbol{\theta} \\ \mathbf{p}_f \\ \mathbf{p}_T \end{bmatrix} = \begin{bmatrix} (\mathbf{Q}_v)_R \\ (\mathbf{Q}_v)_\theta \\ (\mathbf{Q}_v)_f \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} (\mathbf{Q}_e)_R \\ (\mathbf{Q}_e)_\theta \\ (\mathbf{Q}_e)_f \\ (\mathbf{Q}_e)_T \end{bmatrix} \tag{3}$$

The final form of equations include equations of motion associated with a flexible body that incorporates thermal deformation and the reduced order heat equations describing the transient change in the temperature over the flexible body. Thus, the inertia coupling of the reference motion and the thermal deformation is automatically incorporated using the floating frame of reference formulation [2]. Both equations are integrated forward in time simultaneously using general multibody dynamics computer algorithms to account for the coupled structural and thermal behavior of flexible multibody systems.

It is demonstrated by numerical examples that thermomechanical coupling modes induced by external heat flux and prescribed surface temperature are well captured through the thermomechanical constraint modes with appropriate selection of interface thermal coordinates to assure accuracy with the minimum set of fixed-interface truncated thermomechanical modes. The spatial slider crank mechanism, shown in Figs 2 and 3, is used to demonstrate the thermomechanical simulation capability integrated into the general multibody dynamics computer algorithm.

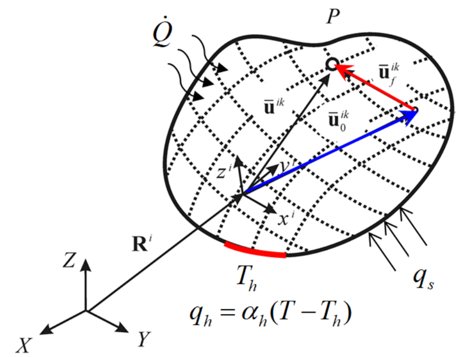


Fig. 1: Thermomechanical coupling model using the floating frame of reference formulation

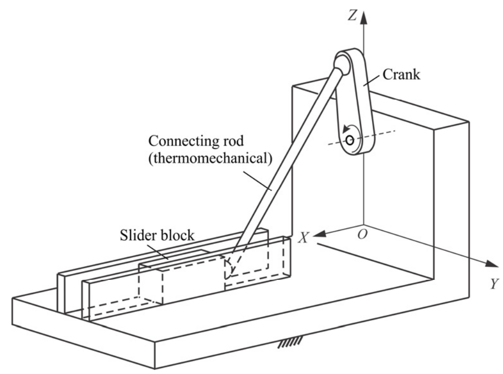
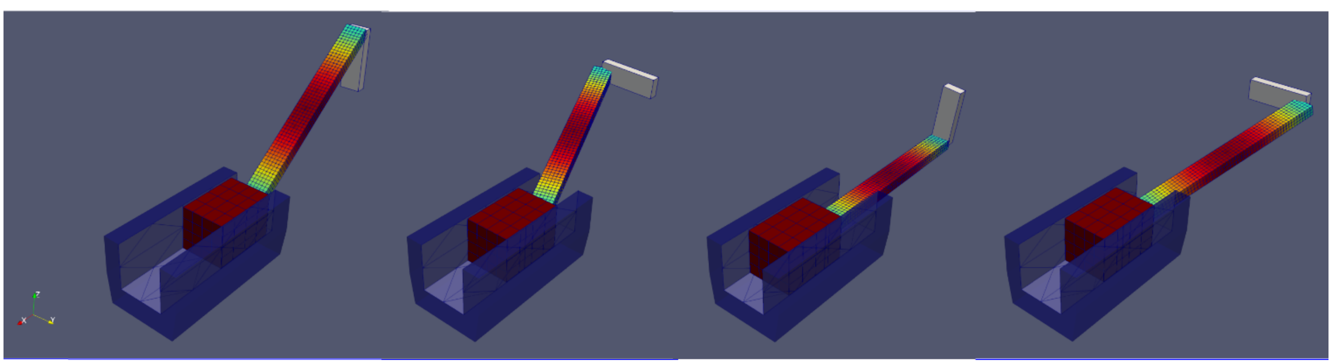


Fig. 2: Spatial slider crank mechanism with thermomechanical connecting rod model



(a) θ=0 deg (b) θ=π/2 (c) θ= π (d) θ= 3π/2

Fig. 3: Spatial slider-crank mechanism with connecting rod subjected to prescribed surface temperature (contour plot indicates temperature distribution)

References

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