Impacts in case of triple unilateral constraint system

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In the paper, the main attention is focused on some modeling method used to model phenomena appearing in impacts, where the last are present between elements of a multibody system and the reference body being interpreted as the motionless ground. Impacting bodies are considered as relatively rigid elements. As its consequence, all impact periods are considered as infinitesimally short, (i.e., their durations as negligible in compare to the other integrated periods). It is not a novel aspect, but in the previously presented publications [1, 2, 3], analyses were restricted to a single impact point present between the selected bodies of the considered system, only. Other constraints (when present in the system) were considered as bilateral. In the present test, a triple unilateral contact (points *A*,*B*,*C* in Fig.1a) is considered. The main body of the system is at rest, supported by two unilateral contacts (points *A* and *B*). As mass of the main body is relatively high, these two contacts are preserved during all the pre-impact period of calculation. An additional arm is attached to the main body and it rotates at a high speed. It impacts to the ground in the third point (point *C*) of the triple unilateral contact.

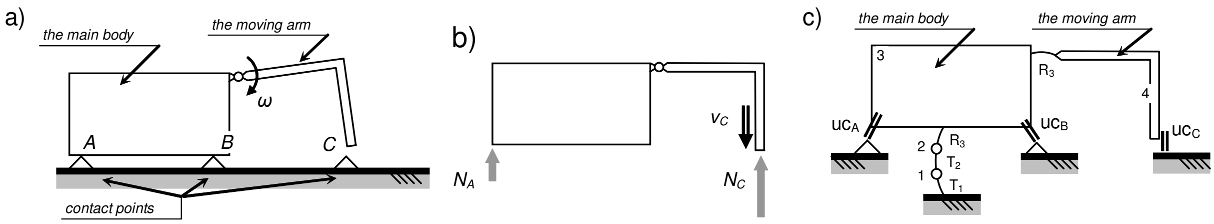

Fig. 1: Considered system: sketch of its physical model (a); considered multibody structure (b)

According to the classical multibody modeling, as presented in [2], dynamic model of the system is written in the well known matrix form:

$$\boldsymbol{M}(\boldsymbol{q})\cdot\ddot{\boldsymbol{q}}+\boldsymbol{F}(\dot{\boldsymbol{q}},\boldsymbol{q})+\boldsymbol{Q}(\dot{\boldsymbol{q}},\boldsymbol{q},t)=0 \tag{1}$$

where: $\boldsymbol{q}$ - col. matrix of joint coordinates; $\boldsymbol{M}$ - mass matrix; $\boldsymbol{F}$ - vector of generalised forces set form centrifugal, gyroscopic and Coriolis terms; $\boldsymbol{Q}$ - vector of generalised joint forces; t - time.

When unilateral contacts are introduced, additional contact forces $\boldsymbol{N}$ (Fig. 1.b) have to be considered. It leads to:

$$\boldsymbol{M}(\boldsymbol{q},t)\cdot\ddot{\boldsymbol{q}}+\boldsymbol{F}(\dot{\boldsymbol{q}},\boldsymbol{q})+\boldsymbol{Q}(\dot{\boldsymbol{q}},\boldsymbol{q},t)+\boldsymbol{J}^T(\boldsymbol{q})\cdot\boldsymbol{N}\ =0 \tag{2}$$

To obtain the requested time evolutions of the velocities, the introduced dynamics equations have to be integrated. However, as it was shown in some of the previous publications [1-2], there is a significant list of assumptions that effect to vital numerical simplifications of this process. Firstly, with finite velocities of the considered bodies, all the potentially possible changes are negligible small for the joint positions in the infinitesimal period of the impact. By contra, the velocity changes are finite. They arise as some results of forces infinite in magnitudes acting during the infinitesimally short periods of impact. Secondly, some of the integrals are infinitesimal and can be neglected (e.g., integrals of the $\boldsymbol{F}$ and $\boldsymbol{Q}$ vectors). Thirdly, detail knowledge of the time evolution of the contact forces is a secondary aspect. It is the time integrals (called the *impacts of the force*) that play the important role in the calculation. What is more, the time integration is not necessary to evaluate the total net value of these integrals. Whatever are the introduced elastic properties of the impacting regions, the impacts of the forces can be evaluated on the base of algebraic equations that correlate dynamic of the system, initial relative velocities of the impacting elements and the restitution coefficients, as well. Summarising these properties, the dynamics equation can be written as:

$$\boldsymbol{M}(\boldsymbol{q},t)\cdot\Delta\dot{\boldsymbol{q}}+\boldsymbol{J}^T(\boldsymbol{q})\cdot\boldsymbol{I}_c=0 \quad : \quad \Delta\dot{\boldsymbol{q}}=\int_t^{t+\Delta t}\ddot{\boldsymbol{q}}\cdot dt \quad ; \quad \boldsymbol{I}_c=\int_t^{t+\Delta t}\boldsymbol{N}\cdot dt \;, \tag{3}$$

Non-dissipative impacts are considered in the model. According to it, two separate sub-periods of the impact has to be considered: the compression period and the expansion period. The expansion impacts are modeled by some restitution coefficients. When focusing on details of the considered process, the impulsive force at point *C* is affecting the arm kinematics and the kinematics of the rest of the bodies of the system, as well. Infinite accelerations are detected in the system, as well as impacts in the other contact points. The two contact points of the main body (points *A* and *B*) behave differently. At the first one (point *B*), positive velocity changes are detected starting from the beginning of the period of the impact (Fig. 2.b). This contact is lost without any impact forces in its region. At the second point (point *A*), negative velocity changes are present. As a result, there are some compressions of the contacting regions and some additional impulsive force is considered in this contact point (Fig 2.a). When the introduced restitution coefficients are recalled, it results in some positive values of the velocities after the end of the period of the impacts (Fig 2.c).

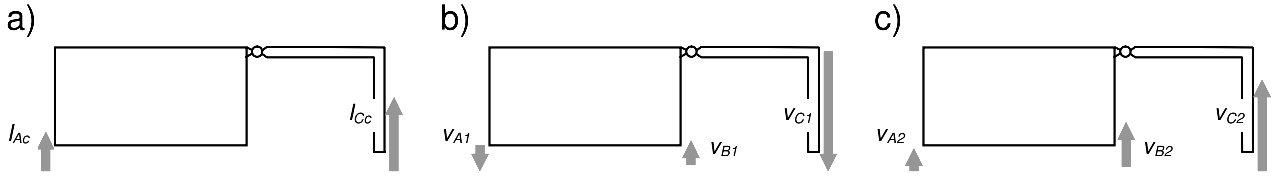

Fig. 2: Impacts and velocities at the impacting points: detected impacts (a) velocities at the initial period of compression (b) velocities at the final period of expansion (c)

The multibody model is introduced as relatively elementary (Fig. 1.b). Its main attention is focused on the impact modeling. Two rigid bodies are considered. They are joined by a revolute joint only (joint R_3 between bodies #*3* and #*4* in Fig. 1.b). The main body (body #*3*) has the free planar mobility. According to it, there is a kinematic chain of three joints that connect it with the reference body. At two lower corners of the body unilateral contact constraints are introduced between the body and the reference. The third unilateral constraint is introduced between the terminal point of the arm (body #*4*) and the reference.

According to some initially formulated hypothesis, it is supposed that detail knowledge of the time evolution of the contact force is not necessary in case of two impacts appearing simultaneously as the fundamentally important parameters are the integrated values only (i.e., the impacts of the forces). The introduced hypothesis is a kind of extinction of the classical conclusions formulated for the single impact cases. Unfortunately, the results obtained with this hypothesis are far from the practice base expectations. To verify it, the obtained results are compared with the alternative ones, where elastic deformations at the contact regions are analyzed with higher details. Performed comparisons have verified that the initially proposed hypothesis is not true, and the multi-impact process is slightly more complex. Final velocities of the system elements depend not only on the initial velocities of the impacting elements, but on the characteristics of the elastic and the damping properties, too. According to it, some modified formula is proposed for the case of the double contacts.

References

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