A DAE approach to an interaction problem between a sloshing and a structural vibration

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This study addresses the coupled behavior between a structure and a liquid surface behavior in a partially filled liquid container (sloshing). It is an important engineering problem for such applications as designs of liquid cargo transportations, tuned liquid dampers (TLDs) [1] and so on. In particular, this paper focuses on the TLDs, which is a kind of dynamic damper for suppressing vibration of a tall building. Many works studied the interaction between the structure with one degree of freedom and one TLD. This model is available to evaluate performance of the TLD for a specific mode structural vibration. Mostly, it applies to the lowest mode in the design of TLD against vibration caused by wind. On the other hand, a structure with multi-degrees of freedom is more reasonable when a matter is an installation position of the TLD.

We consider a two-dimensional problem for the coupling between the nonlinear sloshing and a structure modeled as a lumped mass system with N-DoF shown in Fig. 1. An assumption of inviscid irrotational flow is introduced for liquid in the TLDs. The TLDs have vertical side walls and a flat bottom, which can be regarded as rigid walls. In this study, only masses with the index in the set N_{TLD} are equipped with the TLDs, namely, $m^{(i)}$ $(i \in N_{\mathrm{TLD}})$. The quantities in the fluid domain are described by using the tank-attached Cartesian coordinate system $(o{-}xy)$. The structure is subjected to external base excitation in a horizontal direction. The Lagrangian of this system is given by

$$L = \sum_{I=1}^{N} L^{(S,I)} + \sum_{i \in N_{\mathrm{TLD}}} \{ L^{(F,i)} + \Lambda^{(i)} (\dot{X}^{(i)} - V^{(i)}) \}, \tag{1}$$

$$L^{(S,I)} = \frac{1}{2} M^{(I)} (V^{(I)})^2 - \frac{1}{2} K^{(I)} (X^{(I)} - X^{(I-1)})^2, \quad L^{(F,i)} = \iiint_{\Omega^{(i)}} P^{(i)}(x,y,t)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z, \tag{2}$$

where $\Omega^{(i)}$ is the domain filled with fluid. In order to reduce the order of time derivative of X, the constraint given by the last term in Eq. (1) is introduced. The Lagrangian of fluid shown in the second equation of Eq. (2) is derived by the variational principle in continuum mechanics[2]: the Lagrangian density is equivalent to pressure. This study considers the inviscid and irrotational flow. Thus, the velocity potential $\Phi^{(i)} = \Phi^{(i)}(x,y,t)$ can be introduced. It gives a following expression for the pressure $P^{(i)}(x,y,t)$

$$P^{(i)} = -\rho^{(F)} \left\{ \frac{\partial \Phi^{(i)}}{\partial t} + \frac{1}{2} \nabla \Phi^{(i)} \cdot \nabla \Phi^{(i)} + \frac{1}{2} g y + x \dot{V}^{(i)} \right\}, \tag{3}$$

where $\rho^{(F)}$ is the fluid density. The operator ∇ is defined by $\nabla = (\partial/\partial x,\, \partial/\partial y)^T$. Let $\boldsymbol{Q}$ be vectorial description of generalized coordinates comprised of $\eta^{(i)}$, $\Phi^{(i)}$, $X^{(I)}$ and $V^{(I)}$. It can be found that the Lagrangian given by Eq. (1) has singularities: the matrix derved by $\partial^2 L / \partial \dot{\boldsymbol{Q}} \partial \dot{\boldsymbol{Q}}$ is singular. In order to formulate the system with these singularities, this study introduces the method for a constrained system developed by Dirac [3]. In this method, the singularities are treated as constraints given by

$$\boldsymbol{C}^{(i)} = \left(\pi^{(\eta,i)} - \frac{\delta L}{\delta \dot{\eta}^{(i)}},\ \pi^{(\Phi,i)} - \frac{\delta L}{\delta \dot{\Phi}^{(i)}},\ \pi^{(X,I)} - \frac{\delta L}{\delta \dot{X}^{(i)}},\ \pi^{(V,I)} - \frac{\delta L}{\delta \dot{V}^{(i)}} \right)^T = \boldsymbol{0}, \tag{4}$$

where the functional derivative defined by $\delta F[f(x)]/\delta f(\xi) = \lim_{\varepsilon \to 0} (1/\varepsilon)\{F[f(x) + \varepsilon \delta(x-\xi)] - F[f(x)]\}$ is introduced. These constraints derived by the definition of momentum are referred to as a primary constraint.

In order to derive the Hamiltonian, the Legendre transform is applied to the Lagrangian given in Eq. (1). Then, the Hamiltonian is modified by means of the Lagrange's method of undetermined multiplies with the constraint shown in Eq. (4), it gives an augmented Hamiltonian as follows:

$$H = T + U + \sum_{i \in N_{\mathrm{TLD}}} \boldsymbol{\lambda}^{(i)} \cdot \boldsymbol{C}^{(i)} \tag{5}$$

$$T = \iiint_{\Omega^{(i)}} \frac{1}{2} \nabla\Phi^{(i)} \cdot \nabla\Phi^{(i)} \mathrm{d}x\mathrm{d}y\mathrm{d}z, \quad U = \iiint_{\Omega^{(i)}} gy \, \mathrm{d}x\mathrm{d}y\mathrm{d}z.$$

The equations of motion of the constrained system with the Hamiltonian (5) can be obtained in the form

$$\dot{\boldsymbol{Q}} = \frac{\delta H}{\delta \boldsymbol{P}}, \quad \dot{\boldsymbol{P}} = -\frac{\delta H}{\delta \boldsymbol{Q}}, \quad 0 = \frac{\delta H}{\delta \boldsymbol{\lambda}^{(i)}} = \boldsymbol{C}^{(i)} \quad (i \in N_{\mathrm{TLD}}). \tag{6}$$

The set of equations given by (6) involves the kinematic condition and the equilibrium of the pressure on the liquid surface, the continuity equation (Laplace's equation) and the boundary conditions on the TLD tank walls. This study applies an asymptotic model and the modal decomposition technique with eigen functions which satisfy Laplace's equation and the boundary conditions on the wall to Eq. (6). It is worth noting that the set of equations given by (6) is classified in the DAEs with index-3. This study employs the direct numerical integration technique based on the Energy-momentum method [4] in order to solve the DAEs.

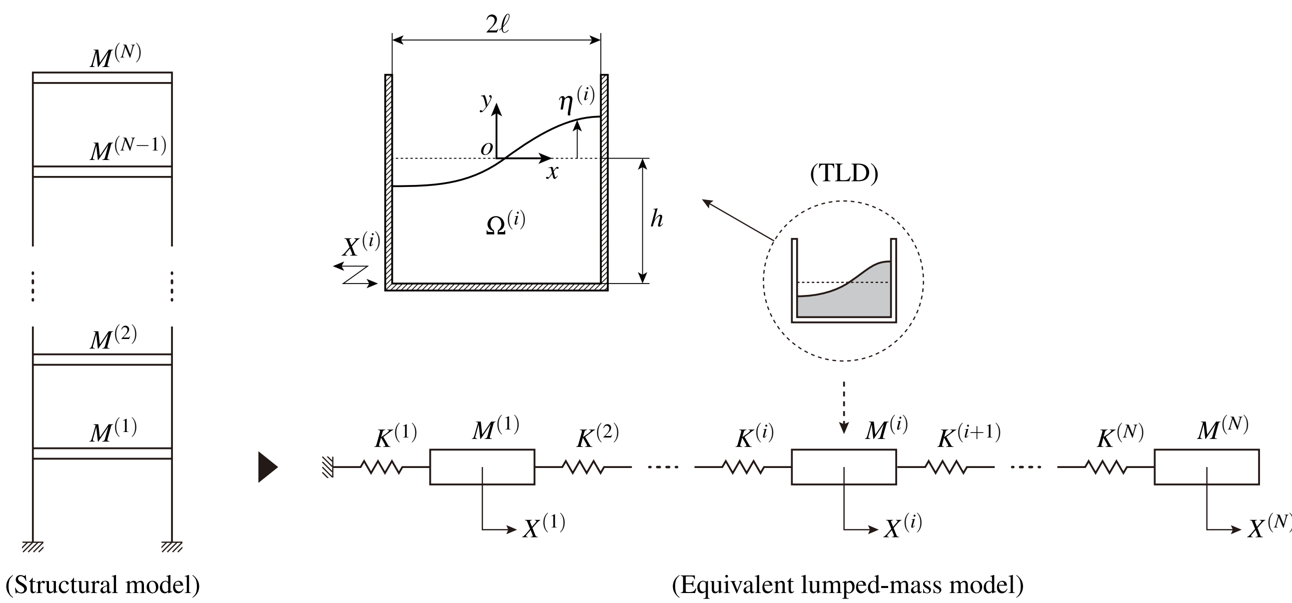

Fig. 1: Analytical model for the coupling problem between TLDs and the structure with N-DoF.

References

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