Flexible multibody dynamics using polygonal elements

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This work presents a computational environment for dynamic simulations of flexible Multibody Systems (MBS) using polygonal finite elements mesh. Recent works in topology optimization ([1], [2], [3]) have shown that polygonal elements can be very useful in numerical analysis since they provide great flexibility in discretizing complex domains and they are computationally very efficient when compared to conventional simplex elements (eg., triangles and quads)[1]. To the best of author's knowledge this kind of elements have not been used before for MBS simulations. Two problems will be addressed: the *first* one consists of two plate bodies in a vertical plane x-y with a point mass in the tip and two rotational joints with z-axis degree of freedom as shown in the Fig. 1a. The *second* problem involves adding angle motors to each joint such that the same time-dependent angle is imposed to each rotational joint.

Both problems comprise the analysis of flexible multibody systems composed of plates in plane stress state, subject to large displacements and small deformations. For simplicity, the material constitutive relationship is chosen as linear. The 2D elements are subject to stretching and bending and the point mass has no rotational effects[2].

The methodology used here to solve these problems follows the variational approach for a constrained MBS as stated in ([5]), ([6]) and ([7]). The flexible MBS can be modeled as a set of discretized bodies represented by a set of n generalized coordinates $\boldsymbol{q}$, satisfying the following set of m holonomic constraints

$$\boldsymbol{\phi}(\boldsymbol{q},t)=\boldsymbol{0}, \tag{1}$$

The Lagrangian function $\mathscr{L}$ that relates the kinetic $\mathscr{K}$, potential $\mathscr{V}$ and the deformation $\mathscr{W}$ energies can be written as

$$\mathscr{L}(\boldsymbol{q},\dot{\boldsymbol{q}})=\mathscr{K}(\boldsymbol{q},\dot{\boldsymbol{q}})-\mathscr{V}(\boldsymbol{q})-\mathscr{W}(\boldsymbol{q}). \tag{2}$$

The vector holding the nodal displacements $\boldsymbol{q}(t)$ is obtained by solving the variational problem

$$\delta\int_{t_1}^{t_2}\left(\mathscr{L}-\boldsymbol{\lambda}^T\boldsymbol{\phi}\right)dt=0. \tag{3}$$

where $\boldsymbol{\lambda}$ is a vector of Lagrange multipliers that take into account the geometrical constraints amongst the bodies. After integrating Eq. (3), a differential-algebraic system of equations (DAE) is obtained as

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{g}^{gyr}(\boldsymbol{q},\dot{\boldsymbol{q}})+\boldsymbol{g}^{int}(\boldsymbol{q})-\boldsymbol{g}^{ext}(\boldsymbol{q})+\boldsymbol{\phi}_{\boldsymbol{q}}^T(\boldsymbol{q},t)\boldsymbol{\lambda}=\boldsymbol{0} \tag{4}$$

$$\boldsymbol{\phi}(\boldsymbol{q},t)=\boldsymbol{0}, \tag{5}$$

where $\boldsymbol{g}^{gyr}=\partial\mathscr{K}/\partial\boldsymbol{q}$, $\boldsymbol{g}^{int}=\partial\mathscr{W}/\partial\boldsymbol{q}$, $\boldsymbol{g}^{ext}=-\partial\mathscr{V}/\partial\boldsymbol{q}$, $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{\phi}_{\boldsymbol{q}}(\boldsymbol{q},t)=\partial\boldsymbol{\phi}/\partial\boldsymbol{q}$ and the force vectors are gathered into $\boldsymbol{g}=\boldsymbol{g}^{gyr}+\boldsymbol{g}^{int}-\boldsymbol{g}^{ext}$.

In order to solve this continuous system of equations, Eq. (4) is expressed as a residual $\boldsymbol{r}=\boldsymbol{M}\ddot{\boldsymbol{q}}+\boldsymbol{g}+\boldsymbol{\phi}_{\boldsymbol{q}}^T\boldsymbol{\lambda}$, whose solution is $\boldsymbol{q}(t)$, $\boldsymbol{\lambda}(t)$. The time is discretized into equal steps and we assume that initial conditions

[1]Moreover, since the next step of the present research is to apply topology optimization techniques to minimize the overall weight of the multibody systems and for this purpose polygonal elements in topology optimization has proven to be ([1], [2], [3]) more efficient when compared to conventional elements and prevents the formation of anomalies such as checker boarding pattern or single node connections, we decided use polygonal elements in this work.

[2]The natural three dimensional extension for the polygonal elements are the polyhedral elements. These 3D elements have successfully been used for solving static problems in the context of topology optimization applications [4].

$q_0, \dot{q}_0, \ddot{q}_0, \lambda_0$ are given. The solver attempts to minimize both r and ϕ, according to a prescribed numerical tolerance and this is accomplished by integrating Eq. (4) using the generalized-α method ([8]) with the Newton linearization scheme. Additionally, it should be noticed that high index DAE numerical solutions are polluted by perturbations due to finite arithmetic precision when small time steps are chosen. Thus a preconditioning scheme is necessary. In this work we use the method described in ([9]) is used.

The methodology used here is implemented in the MATLAB programming environment and the two problems are also validated using the ANSYS Workbench software, for gaining insight and confidence. The deformed plates for the first problem, at two different time steps are shown in Fig. 1b.

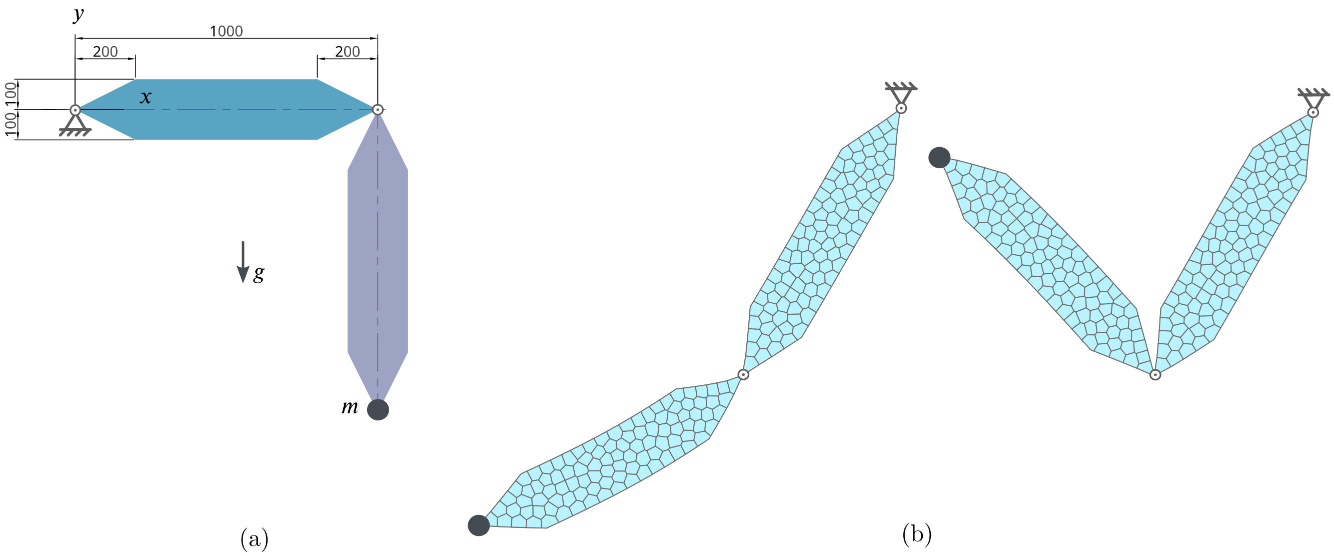

Fig. 1: (a) Two vertical flexible plates with two joints and a mass (units in mm). (b) Deformed plates at two different time steps.

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