Efficient design optimization of beam cross-sections for flexible multibody dynamics

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The design of complex beam-like composite structures such as helicopter rotor blades or wind turbine blades has received considerable attention over the last decades [1, 2]. In addition to the multidisciplinary nature of such problems, finding the most effective and efficient structural dynamics solver is not necessarily straightforward [3]. Traditional beam models are often inaccurate because of the presence of large centrifugal forces, geometric stiffening phenomena, and anisotropic material configurations. On the other hand, full three-dimensional finite-element methods are computationally expensive in a design optimization context, and they require a level of detail that is often unnecessary.

This paper presents a method for the design optimization of two-dimensional beam cross-sections that are defined in a parametric way, which constitutes a form of *shape* and *material optimization*. The resulting cross-sectional properties can later be used within a three-dimensional flexible multibody dynamics solver that uses geometrically-exact beam models. The unique characteristics of this approach are: (1) the Saint-Venant solution of arbitrary cross-sections, which provides detailed warping, stiffness properties and three-dimensional stresses within the cross-section; and (2) the efficient solution of the elasticity and adjoint problems, which provides the design sensitivities that are used by the design optimization module.

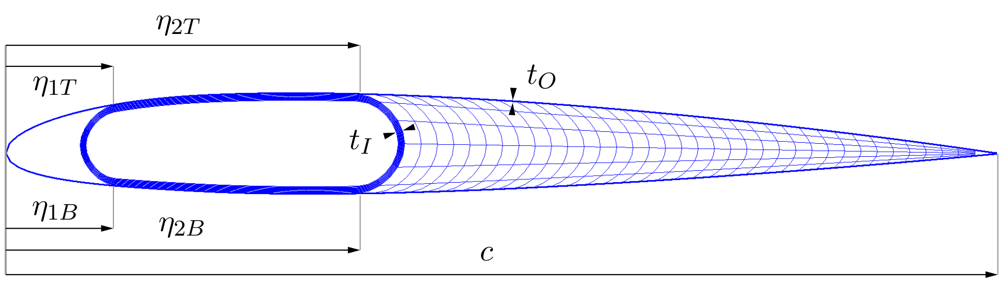

Fig. 1: Example shape parameters in blade cross-section

The use of *topology optimization* methods that allow one to arbitrarily distribute the material throughout the cross-section is very limited within blade cross-section design. The main reason is that these sections are far from homogeneous and the placement of material is subject to numerous constraints, including the outer mold line. In contrast, the present shape and material optimization approach allows the user to define basic design parameters according to cross-section templates, which are often directly related to manufacturing constraints and decades of design experience.

SectionBuilder provides the solution of the linear theory of three-dimensional elasticity, which is in agreement with Saint-Venant's theory for torsion. The algorithm is based on the semi-discretization of the beam, by which the finite element mesh extends only over the cross-section [4]. *SectionBuilder* effectively links global cross-sectional properties such as bending stiffness (EI) and torsional stiffness (GJ) to specific local parameters such as internal geometric dimensions and material properties. The basic structural equations can be summarized as

$$\underline{\underline{A}}\,\underline{\underline{X}}_1 = \underline{\underline{B}}_1, \tag{1a}$$

$$\underline{\underline{A}}\,\underline{\underline{X}}^\dagger = \underline{\underline{B}}_0 - \underline{\underline{C}}\,\underline{\underline{X}}_1, \tag{1b}$$

$$\underline{\underline{W}}^\dagger = \underline{\underline{D}}\,\underline{\underline{X}}^\dagger, \tag{1c}$$

where $\underline{\underline{W}}^\dagger$ constitutes the warping field (equivalent to the displacement vector in a standard static finite-element method procedure). The remaining terms are detailed in [4]. A series of calculations then lead to the compliance and stiffness matrices, namely $\underline{\underline{S}}$ and $\underline{\underline{K}}$.

The adjoint sensitivity analysis [5] provides the derivatives of each component of the compliance (S_{ij}) or stiffness (K_{ij}) matrix with respect to the design parameters $\underline{b}$

$$\frac{\mathrm{d}S_{ij}}{\mathrm{d}\underline{b}} = \frac{\partial S_{ij}}{\partial \underline{b}} + \underline{\Lambda}_1^T \frac{d\underline{\underline{\mathcal{A}}}'}{d\underline{b}} \underline{X}_1 + \underline{\Lambda}_0^T \left(\frac{d\underline{\underline{\mathcal{A}}}'}{d\underline{b}} \underline{X}^\dagger + \frac{d\underline{\underline{\mathcal{C}}}'}{d\underline{b}} \underline{X}_1 \right), \tag{2}$$

where some variables have been vectorized as follows: $\underline{\underline{\mathcal{A}}}' \equiv \left(\underline{\underline{I}}_6 \otimes \underline{\underline{\mathcal{A}}}\right)$, $\underline{\underline{\mathcal{C}}}' \equiv \left(\underline{\underline{I}}_6 \otimes \underline{\underline{\mathcal{C}}}\right)$, $\underline{X}_1 = \mathrm{vec}\left(\underline{\underline{X}}_1\right)$, $\underline{X}^\dagger = \mathrm{vec}\left(\underline{\underline{X}}^\dagger\right)$, $\underline{\mathcal{B}}_1 = \mathrm{vec}\left(\underline{\underline{\mathcal{B}}}_1\right)$, $\underline{\mathcal{B}}_0 = \mathrm{vec}\left(\underline{\underline{\mathcal{B}}}_0\right)$. The adjoint variables $\underline{\Lambda}_0$ and $\underline{\Lambda}_1$ are computed from

$$\underline{\underline{\mathcal{A}}}'^T \underline{\Lambda}_0 + \left(\frac{\partial S_{ij}}{\partial \underline{X}^\dagger}\right)^T = \underline{0}, \tag{3a}$$

$$\underline{\underline{\mathcal{A}}}'^T \underline{\Lambda}_1 + \underline{\underline{\mathcal{C}}}'^T \underline{\Lambda}_0 = \underline{0}. \tag{3b}$$

The remaining terms, namely $\partial S_{ij}/\partial \underline{X}^\dagger$, $\partial S_{ij}/\partial \underline{b}$, $d\underline{\underline{\mathcal{A}}}'/d\underline{b}$ and $d\underline{\underline{\mathcal{C}}}'/d\underline{b}$ must be manually obtained [5].

The proposed objective function is a weighted sum of two non-dimensional metrics, namely a component of the cross-sectional compliance and the mass per unit span

$$g = w_S \left(\frac{S_{ij} - S_{ij}^*}{S_{ij}^*}\right)^2 + w_m \left(\frac{m_{00} - m_{00}^*}{m_{00}^*}\right)^2, \tag{4}$$

where $(^*)$ denotes the value obtained when the variable is individually minimized. The weighting factors are calculated from

$$w_S \left(\frac{S_{ij}^\circ - S_{ij}^*}{S_{ij}^*}\right)^2 = \frac{1}{2}, \quad w_m \left(\frac{m_{00}^\circ - m_{00}^*}{m_{00}^*}\right)^2 = \frac{1}{2}. \tag{5}$$

where $(^\circ)$ denotes the baseline value.

A number of examples have been analyzed to demonstrate the use of the presented approach, including rectangular cross-sections, C-sections and helicopter blade-type cross-sections. A gradient-based optimization method drives the design process. Every design iteration requires the computation of the adjoint variables and the design sensitivities, which are used to reduce the objective function, namely the bending and torsional compliance of the cross-section. The results are validated using complex- and real-step numerical differentiation. Results show that the adjoint method provides machine-precision sensitivities, which in turn increases the performance of the optimization process.

References

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