Superelements in a minimal coordinates floating frame of reference formulation

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In many cases, multibody dynamics problems are concerned with systems in which large relative rotations between bodies are the primary source of nonlinearities. That is, the elastic deformation of each individual body can often be considered to be small relative to a local frame moving along with the body. The floating frame of reference formulation is a commonly used well-developed formulation for such multibody systems. In this formulation, the generalized coordinates of a flexible body consist of the position and orientation of the floating frame of reference and the generalized coordinates describing the body's local elastic displacement field. Kinematic constraints between bodies are enforced by Lagrange multipliers, which increases the total number of unknowns in the system's constrained equations of motion.

An important advantage of the floating frame of reference formulation is the possibility to describe the flexible behavior of individual bodies by their linear finite element models. In fact, the mass and stiffness matrices created in a finite element analysis of a single body can be reused in the multibody analysis of the entire system. This allows for the use of substructuring methods in the floating frame of reference formulation. Using model order reduction techniques, the large finite element models of individual bodies can be reduced to a limited number of generalized coordinates, corresponding to the flexible mode shapes of the reduction basis.

In order to obtain the system's equations of motion in terms of a minimal set of coordinates, the Lagrange multipliers need to be eliminated from the formulation. This can be done if the absolute motion of coordinate systems attached to the flexible body's interface points uniquely describes the body's global kinematics. One could say that in this case a so-called superelement is created: the motion of the flexible body is described entirely by the motion of its interface points. This is similar to that the displacement field of a finite element is described uniquely by the displacements of its nodes. Hence, if it is possible to express both the floating frame coordinates and the local elastic degrees of freedom together in terms of the absolute interface coordinates, the Lagrange multipliers can be eliminated.

In previous works, several methods for creating such superelements have been proposed. These methods require the floating frame to be at an interface node [1] or express the location and orientation of the floating frame as a weighted average of the interface coordinates [2]. In the first method, simulation results are dependent on the interface point chosen, which introduces an unnecessary discrimination between interface points. Moreover, better accuracy is obtained if the floating frame is close to the body's center of mass. The second method does not suffer from these drawbacks, but because the floating frame is not attached to a material point of the body, its motion cannot be given more physically interpretation than that it somehow represents the body's average rigid body motion.

In [3], a new method is proposed that enables a coordinate transformation to absolute interface coordinates in such a way that the floating frame can be attached to a material point, not being an interface point. To this end, the Craig-Bampton method is applied to reduce the size of system matrices that describe the elastic behavior of the flexible bodies. The Craig-Bampton interface modes are then used as a reduction basis to obtain the local mass and stiffness matrices. In this, the standard floating frame of reference formulation yields a model with the 6 absolute coordinates of the floating frame plus the $6N$ local coordinates corresponding to the Craig-Bampton modes of the N interface points. Since the idea is to express the kinematics of the body in terms of the $6N$ absolute interface coordinates only, the coordinates of the floating frame must be eliminated and the local interface coordinates and the absolute interface coordinates have to be related. This is possible by imposing 6 conveniently chosen additional constraints.

If the floating frame would be located at an interface point, the Craig-Bampton modes of the corresponding interface point can simply be removed [1]. If the floating frame coordinates are an explicit weighted average of the absolute interface coordinates, this would make the absolute interface coordinates the independent generalized coordinates only [2]. In the new method, the floating frame is attached to a convenient material point. The material point that coincides with the center of mass of the undeformed body would be a logical choice. The essence of the new method is that it demands that the local elastic deformation at the location of the floating frame is zero [3]. That is, the Craig-Bampton modes are determined without a priori knowledge of the location of the floating frame, but any linear combination of Craig-Bampton modes should be such that there is no deformation at the floating frame. This introduces 6 constraints from which it is possible to express the absolute floating frame coordinates and the local interface coordinates in terms of the absolute interface coordinates.

The strength of this new method is that it enables the construction of superelements, suitable for the use in the floating frame of reference formulation in a very general way. Moreover, it will be shown that this approach has striking similarities with the way the corotational frame is defined based on the absolute nodal coordinates of a finite element in the corotational finite element formulation. However, within this framework, the new method can be seen as a generalization of current available methods: It can be applied on any arbitrarily shaped body with an arbitrary number of interface points, whereas similar methods in the corotational frame formulation are dedicated to specific types of elements (in particular beams).

In this work, the mathematical elaborations required for the kinematic transformations will be presented with emphasis on the physical interpretation of the many terms in the transformation matrices. Also, it will be shown that the new methods yields reliable results when it is validated with a wide variety of benchmark problems.

[1] A. Cardona and M. Géradin, "Modeling of superelements in mechanism analysis," *International Journal for Numerical Methods in Engineering*, vol. 32, pp. 1565–1593, 1991.

[2] A. Cardona, "Superelements Modelling in Flexible Multibody Dynamics," *Multibody System Dynamics*, vol. 4, pp. 245–266, 2000.

[3] M.H.M. Ellenbroek and J.P. Schilder, "On the unse of absolute interface coordinates in the floating frame of reference formulation for flexible multibody dynamics," *Multibody System Dynamics*, accepted for publication on 01-12-2017.