Comparison of motion representations for efficient numerical simulation of flexible multibody systems

Valentin Sonneville and Olivier A. Bauchau
Department of Aerospace Engineering, University of Maryland
{vspsonn,obauchau}@umd.edu

The non-vectorial nature of the configuration space of multibody systems allows for several equivalent kinematic representations, see e.g. [1, 2, 3, 4, 5, 6]. For instance, a rotation can be represented equivalently by a 3×3 orthogonal matrix or a unit quaternion. Similarly, a motion can be represented by a pair 3×3 orthogonal matrix - displacement vector, or a unit dual-quaternion. Researchers have adopted one representation over the others typically because they find it more insightful into the kinematics and more convenient to carry out theoretical developments. When it comes to the numerical implementation, a more objective measure of the merit of a representation can be assessed based on the computational efficiency, including the storage and the number of elementary operations.

One of the most basic operation to be performed for kinematics is the composition of motions. As an introductory example to the problem of efficiency, consider the composition of rotations. If the 3×3 orthogonal matrix representation, denoted $\underline{\underline{R}}$, is adopted, the composition is the matrix product: 9 numbers are needed for the storage of each matrix and the composition of rotations requires 45 operations (3 multiplications and 2 additions for each component of $\underline{\underline{R}}_3$). In contrast, the unit quaternion representation, denoted $\underline{p}$ requires 4 numbers for storage and the composition of rotations, i.e $\underline{p}_3 = \underline{p}_2 \circ \underline{p}_1$, can be done with 28 operations (4 multiplications and 3 additions for each component of $\underline{p}_3$). The parametrization of motions is also an important operation in multibody codes and it is affected by the kinematic representation. Consider, for instance, extracting the rotation operator from the 3 Cayley parameters $\underline{a}$. The computation of the 3×3 orthogonal matrix is given by $\underline{\underline{R}} = (\underline{\underline{I}} + \tilde{a})(\underline{\underline{I}} - \tilde{a})^{-1} = \underline{\underline{I}} + \alpha\tilde{a} + \alpha\tilde{a}^2$, where $\alpha = 2/(1+\underline{a}^T\underline{a})$ and $\tilde{a}$ is the skew-symmetric matrix built on the components of $\underline{a}$. The cost of this evaluation can be optimized to 35 operations. The equivalent unit quaternion representation is obtained as $\underline{p} = \alpha/2\,[1;\underline{a}]$, which only requires 11 operations. The unit quaternion representation is thus computationally more efficient for the composition and Cayley-parametrization of rotations. Nevertheless, other operations involving rotations might be used in a general purpose code. For instance, consider the rotation of a three-dimensional vector $\underline{x}$. With the matrix representation, this operation is straightforward, $\underline{\underline{R}}\underline{x}$, and can performed with 15 operations (3 multiplications and 2 additions for each component of the rotated vector). The unit quaternion representation, however, is more complicated, requires intermediate variables and more operations (an optimized implementation would use 3 intermediate numbers and 51 operations). Table 1 summarizes the discussion above. The choice of a representation towards an efficient implementation depends on the type and frequency of operations needed in typical numerical simulations.

Representation	Composition of rotations	Cayley map	Rotation of a vector
3×3 orthogonal matrix $\underline{\underline{R}}$	45	35	15
unit quaternion $\underline{p}$	28	11	51

Tab. 1: Number of operations

In this work, we propose to study several representations of rotations and motions and report their computation efficiency for the simulation of realistic flexible multibody systems. In the pursuit of computation efficiency, the current work relies on a local frame, global parametrization-free framework that the authors and their co-workers have been developing[7, 8, 9, 10, 11]. A key aspect of this framework is the reduction of kinematic non-linearities affecting the equilibrium equations. The most basic operations to be performed are the compositions of motions and the local parametrizations. Indeed, such operations are needed to compute the element matrices relative motions in kinematic joints and flexible joints. In addition to the composition of motions, the time integration method and the finite element discretization require a local parametrization of the motion increments and the relative motions within the elements, respectively.

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