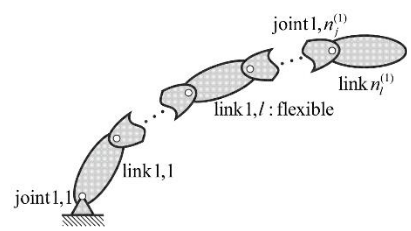
Mathematical modelling of spatial linkages with clearance, friction and links' flexibility effects

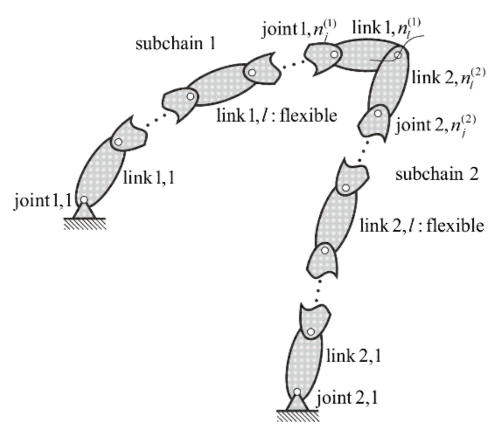
Krzysztof Augustynek, Andrzej Urbaś

Department of Mechanical Engineering Fundamentals, University of Bielsko-Biala, aurbas@ath.bielsko.pl

The paper presents mathematical model of the spatial linkages with flexible links, clearance and friction effects in joints. In the general case kinematic structure the linkage can be open- or closed-loop. Open-loop systems can form serial or tree structure. When closed-loop kinematic chains are analysed, it is necessary to convert such a system into an equivalent system with an open-loop kinematic structure. In the paper the cut-joint technique is used in order to make such conversion. As a result, the spanning tree thus obtained can also have a serial or tree structure. The joint coordinates and homogeneous transformation matrices are used. The flexibility of links is modelled by means of the Rigid Finite Element Method (RFEM) [1]. These approaches differ in the number of degrees of freedom and the way of describing the motion of the rigid finite element. In the modified approach only bending and torsional vibrations of the flexible link are possible to analyse. The classical approach of the RFEM is applied, when you need to analyse influence of all possible deformations of the flexible link (stretching, shear, bending and torsion).

In the paper it is assumed that joints are imperfect i.e. the clearance and friction in joints are taken into account. Bristles' friction models such as the Dahl, LuGre, elasto-plastic or Gonthier models are used. Studies [2] indicate that these models give results close to the experiment and allow to take into account such effects as pre-sliding motion, stick-slip motion and friction-displacement curve hysteresis loop. In addition, LuGre, elasto-plastic or Gonthier models give the possibility to model the Stribeck effect and viscous friction. The clearance in the joints of linkages causes additional vibrations, noise, wear of parts and can lead to damage to the system. It is assumed that the clearance exists only in rotational joints, and two models of the clearance, planar and spatial, are proposed. In the case of the planar model, it is assumed that the axes of the connected parts are parallel, and the contact forces acting between the contacting surfaces are the same along the connection axis. In the case of a spatial model, skewing of the axis additionally is taken into account, which can result in a different distribution of the contact forces along the connection axis [3]. In the paper the influence of the axial clearances are neglected.





The dynamics equations of motion of the linkage are derived using the Lagrange's equations of the second kind. These equations are supplemented with closing constraints equations, formulated for joints in which closed-loop chains are cut.

Depending on the kinematic structure of linkage, the final dynamics equation can take one of the following forms (Fig. 1):

- the open-loop serial structure

$$\mathbf{M}^{(1)}\ddot{\mathbf{q}}=\mathbf{f}^{(1)} \tag{1}$$

- the closed-loop serial structure

$$\begin{bmatrix} \mathbf{M}^{(1)}+\mathbf{M}^{(2)} & -\mathbf{D}^T \\ \mathbf{D} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^{(1)}+\mathbf{f}^{(2)} \\ \mathbf{c} \end{bmatrix} \tag{2}$$

- the open-loop tree structure

$$\left(\mathbf{M}^{(1)}+\ldots+\mathbf{M}^{(s)}+\ldots+\mathbf{M}^{(n_c)}\right)\ddot{\mathbf{q}}= =\left(\mathbf{f}^{(1)}+\ldots+\mathbf{f}^{(s)}+\ldots+\mathbf{f}^{(n_c)}\right) \tag{3}$$

- the closed-loop tree structure

$$\begin{bmatrix} \mathbf{M}^{(1)}+\ldots+\mathbf{M}^{(s)}+\ldots+\mathbf{M}^{(n_c)} & -\mathbf{D}^T \\ \mathbf{D} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = =\begin{bmatrix} \mathbf{f}^{(1)}+\ldots+\mathbf{f}^{(s)}+\ldots+\mathbf{f}^{(n_c)} \\ \mathbf{c} \end{bmatrix} \tag{4}$$

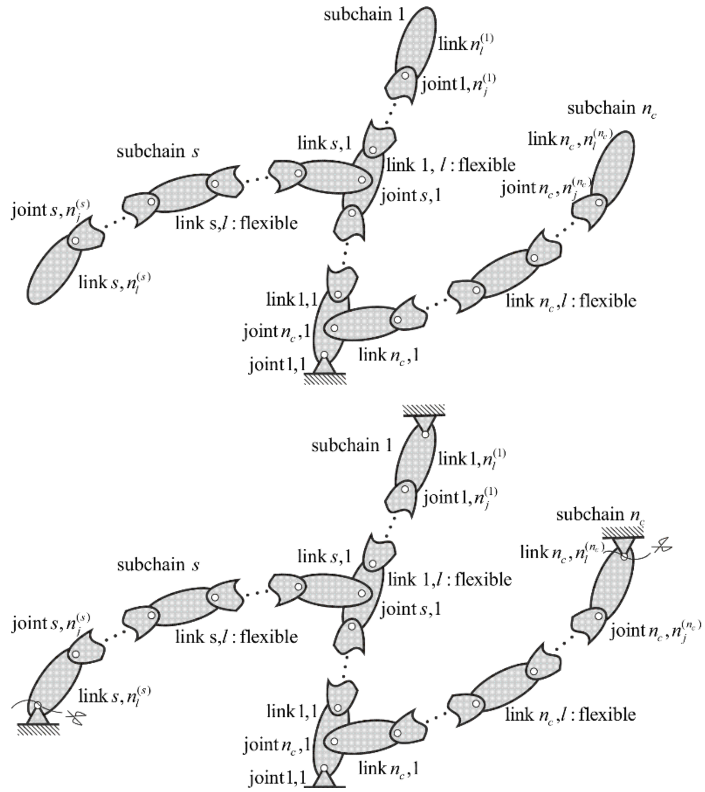


Fig. 1: Possible topologies of linkage

where:
$$\mathbf{M}^{(1)}=\begin{bmatrix} \mathbf{M}_{1,1}^{(1)} & \ldots & \mathbf{0} & \ldots & \mathbf{0} \\ \vdots & \ddots & & & \\ \mathbf{0} & \ldots & \mathbf{0} & \ldots & \mathbf{0} \\ \vdots & & & \ddots & \\ \mathbf{0} & \ldots & \mathbf{0} & \ldots & \mathbf{0} \end{bmatrix}, \mathbf{M}^{(s)}=\begin{bmatrix} \mathbf{M}_{1,1}^{(s)} & \ldots & \mathbf{M}_{1,s}^{(s)} & \ldots & \mathbf{0} \\ \vdots & \ddots & & & \\ \mathbf{M}_{1,s}^{(s)} & \ldots & \mathbf{M}_{s,s}^{(s)} & \ldots & \mathbf{0} \\ \vdots & & & \ddots & \\ \mathbf{0} & \ldots & \mathbf{0} & \ldots & \mathbf{0} \end{bmatrix}, \mathbf{M}^{(n_c)}=\begin{bmatrix} \mathbf{M}_{1,1}^{(n_c)} & \ldots & \mathbf{0} & \ldots & \mathbf{M}_{1,n_c}^{(n_c)} \\ \vdots & \ddots & & & \\ \mathbf{0} & \ldots & \mathbf{0} & \ldots & \mathbf{0} \\ \vdots & & & \ddots & \\ \mathbf{M}_{1,n_c}^{(n_c)} & \ldots & \mathbf{0} & \ldots & \mathbf{M}_{n_c,n_c}^{(n_c)} \end{bmatrix},$$

$$\mathbf{f}^{(1)}=\begin{bmatrix} \mathbf{f}_1^{(1)^T} & \ldots & \mathbf{0} & \ldots & \mathbf{0} \end{bmatrix}^T, \qquad \mathbf{f}^{(s)}=\begin{bmatrix} \mathbf{f}_1^{(s)^T} & \ldots & \mathbf{f}_s^{(s)^T} & \ldots & \mathbf{0} \end{bmatrix}^T, \qquad \mathbf{f}^{(n_c)}=\begin{bmatrix} \mathbf{f}_1^{(n_c)^T} & \ldots & \mathbf{0} & \ldots & \mathbf{f}_{n_c}^{(n_c)^T} \end{bmatrix}^T,$$

$$\mathbf{q}=\begin{bmatrix} \mathbf{q}^{(1)^T} & \ldots & \mathbf{q}^{(s)^T} & \ldots & \mathbf{q}^{(n_c)^T} \end{bmatrix}^T.$$

In numerical simulations, the dynamics of the one-dof spatial linkage are examined. During the simulations, the influence of the clearance and friction on the motion of the linkage's links as well as time courses of the joint forces are analysed. In the full text of the paper results are presented and discussed.

References

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[3] F. Marques, F. Isaac, N. Dourado and P. Flores, "An enhanced formulation to model spatial revolute joints with radial and axial clearances," *Mechanism and Machine Theory*, 116, pp. 123-144, 2017.