# Time integration of nonsmooth mechanical systems using constraints at position, velocity and acceleration levels

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The nonsmooth contact dynamics method was proposed by Moreau and Jean [1, 2] for the simulation of mechanical systems with contact conditions represented by the Signorini law and Coulomb friction models. This time stepping method does not synchronize its time steps to the discontinuous events such as impacts or stick-slip transitions. It can thus efficiently deal with systems with frequent events as it does not involve a detection of each event followed by an integration restart procedure. Also, the nonsmooth contact dynamics method is based on the formulation of the equations of motion in terms of impulses with all bilateral and active unilateral constraints formulated at velocity level. Thus, the impact law is naturally embedded in the expression of the constraints at velocity level. A time discretization according to the backward Euler scheme or to the theta method then leads to robust solvers which are, however, limited to first order accuracy and which satisfy the constraints only approximately at position level.

Recent research efforts in numerical methods for nonsmooth systems aimed at improving the accuracy of the simulation, especially in the presence of elastic bodies, and to enforce the constraints at position-level. For example, the nonsmooth generalized-$\alpha$ method [3] is based on a separation of smooth and nonsmooth contributions in the equations of motion, so that the smooth contributions can be integrated with a higher accuracy. Inspired by the Gear-Gupta-Leimkuhler approach, this algorithm also leads to a numerical solution which exactly satisfies the impact law together with the non-penetration condition without any drift-off effect. The bilateral constraints are also verified both at position and velocity levels. The smooth and nonsmooth motions are nevertheless inherently coupled and the numerical solution can be affected by spurious numerical oscillations of the bilateral reaction forces after the impacts.

In this work, we show that these numerical oscillations can be fully eliminated provided that the equations defining the smooth motion rely on a formulation of the constraints at acceleration level with a suitable activation strategy, see also [4]. At every time step, the algorithm thus enforces the bilateral and active unilateral constraints at position, velocity and acceleration levels. It is shown that stable numerical solutions can be obtained for systems with bilateral and unilateral constraints even if no numerical dissipation is introduced in the generalized-$\alpha$ method.

In order to illustrate the behavior of this algorithm, several numerical examples of smooth and nonsmooth mechanical systems will be presented in the talk. In particular, for the bouncing rigid pendulum shown in Fig. 1 and modelled as a system with both bilateral and unilateral constraints, the numerical solution is computed using two different algorithms:

- the “PVV-constrained” algorithm comes from [3] and enforces the constraints at position and velocity levels;

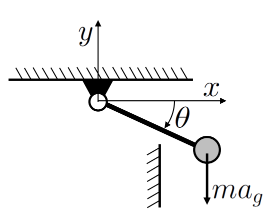


Fig. 1: Bouncing pendulum.

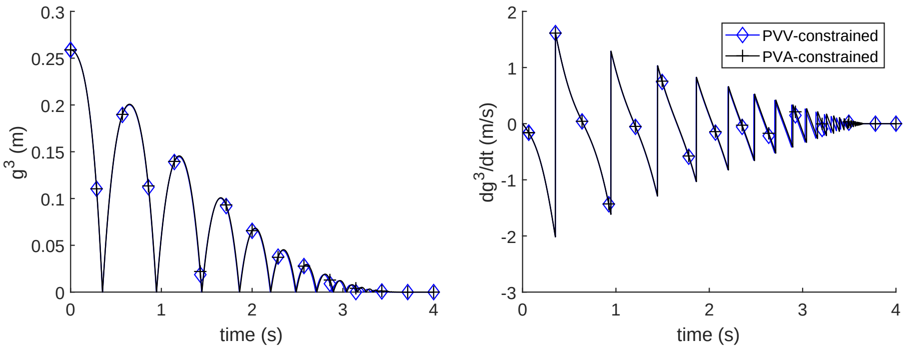


Fig. 2: Unilateral constraint in the bouncing rigid pendulum example (left: position level, right: velocity level).

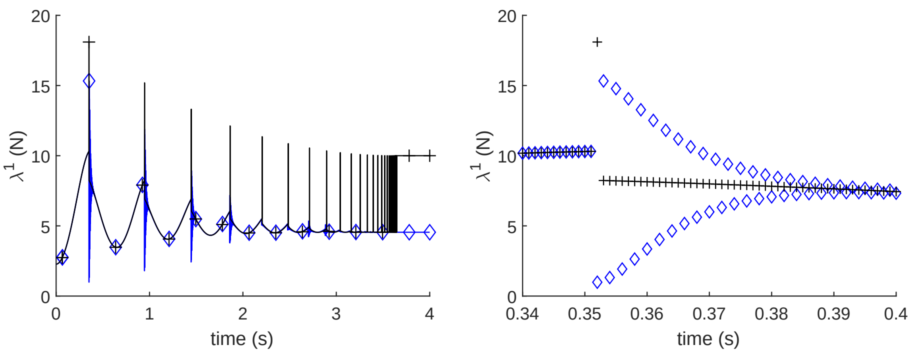


Fig. 3: Smooth bilateral Lagrange multiplier $\tilde{\boldsymbol{\lambda}}$ - left: full time interval, right: zoom on the first impact.

- the "PVA-constrained" algorithm enforces the constraints at position, velocity and acceleration levels.

As can be seen in Fig. 2, the system undergoes impact phenomena with velocity jumps. Every impact also induces an impulse in the bilateral reaction forces. According to the nonsmooth generalized-$\alpha$ method, the reaction forces are splitted into smooth and nonsmooth contributions. The smooth contribution is shown in Fig. 3. The PVV-constrained solution exhibits strong but transient numerical oscillations after every impact, which are damped out by the numerical scheme. In contrast, every impact induces a discontinuity in the PVA-constrained solution but no artificial numerical oscillation is observed in this case.

# References


[1] J.-J. Moreau, "Unilateral contact and dry friction in finite freedom dynamics," in *Non-smooth Mechanics and Applications* (J.-J. Moreau and P. Panagiotopoulos, eds.), vol. 302, pp. 1–82, Wien - New York: Springer-Verlag, 1988.

[2] M. Jean, "The non-smooth contact dynamics method," *Computer Methods in Applied Mechanics and Engineering*, vol. 177, pp. 235–257, 1999.

[3] O. Brüls, V. Acary, and A. Cardona, "Simultaneous enforcement of constraints at position and velocity levels in the nonsmooth generalized-$\alpha$ scheme," *Computer Methods in Applied Mechanics and Engineering*, vol. 281, pp. 131–161, 2014.

[4] O. Brüls, V. Acary, and A. Cardona, "On the constraint formulation in the nonsmooth generalized-$\alpha$ method," in *Advanced Topics in Nonsmooth Dynamics, Transactions of the European Network for Nonsmooth Dynamics* (R. Leine, O. Brüls, and V. Acary, eds.), (Dordrecht, The Netherlands), Springer, 2019.