A Computationally Efficient Approach for Monolithic Simulation of Multibody and Hydraulic Dynamics

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The multibody based real-time simulation approaches allow description of complex products, such as mobile machinery, with a high level of detail for multiple purposes. A dynamic system rarely consists of mechanical components alone, since the mechanical components are often accompanied by hydraulically driven actuator systems. As the hydraulic actuator system has its own dynamic behavior which significantly affects the full system behavior, multibody equations alone are not sufficient to describe the dynamics of the full system. For a more appropriate and robust approach, accurate and effective descriptions must be formed for all dynamic subsystems.

Efficient formulations for both multibody and hydraulic dynamics alone can be found from the literature. A potential candidate for real-time multibody simulation is the semi-recursive method that utilises a penalization scheme [1], similar to the augmented Lagrangian [2]. This approach is shown to be computationally efficient and can lead to robust analysis as it can handle singular configurations and numerical stiffness. To enforce constraints in velocity and acceleration levels the method uses mass-orthogonal projections [3], and a trapezoidal rule for integration. Hydraulic dynamics, in turn, can be described with the lumped fluid method [4] that splits the system into separate volumes that are assumed to have internally constant pressures.

Assuming the equations for both dynamic subsystems are available in the same simulation environment, as seen in certain simulator solutions [5], the monolithic, also known as unified approach is a potential candidate for solving the multiphysics problem at hand. This strongly coupled approach yields a single set of equations to be integrated.

The objective of this research is to introduce a monolithic formulation that uses the semi-recursive formulation and the lumped fluid method for combined real-time simulation of multibody and hydraulic dynamics. To improve generality, and due to complicated forms analytical expressions of the pressure and force descriptions may take, a numerically obtained tangent matrix is used in the Newton's iteration during the integration process. This is in contrast to a similar approach presented in [6] and to most research regarding the semi-recursive formulation. To evaluate efficiency of the proposed method, a similar approach presented by Naya et al. [6] is implemented, with the proposed modification to the tangent matrix, as an efficiency benchmark for the proposed method.

In case of the combined multibody and hydraulic dynamics, the force vector $\mathbf{Q}$ of the semi-recursive method can be written as a function of the joint coordinates $\mathbf{z}$, their first time derivatives $\dot{\mathbf{z}}$, and pressures $\mathbf{p}$. Pressure variation equations can be written according to the lumped fluid method, and also in terms of the same variables. Thus, the following system of equations can be used to describe the full system dynamics:

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{z}} + \mathbf{\Phi}_{\mathbf{z}}^{\mathrm{T}}\alpha\mathbf{\Phi} + \mathbf{\Phi}_{\mathbf{z}}^{\mathrm{T}}\boldsymbol{\lambda}^{*} &= \mathbf{Q}(\mathbf{z},\dot{\mathbf{z}},\mathbf{p}) \\ \dot{\mathbf{p}} &= \mathbf{h}(\mathbf{z},\dot{\mathbf{z}},\mathbf{p}) \end{aligned} \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{\Phi}$ and $\mathbf{\Phi}_{\mathbf{z}}$ are the constraint vector and its Jacobian, correspondingly, α is the penalty factor, $\boldsymbol{\lambda}^{*}$ is the vector of iterated Lagrangian multipliers, and $\mathbf{h}$ are the pressure variation equations.

After applying the difference equations of the trapezoidal rule to Eq. (1), dynamic equilibrium at time step $n+1$ can be obtained. The well known Newton-Raphson iteration can be applied to solve the nonlinear system of equations:

$$\left[\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}}\right]_{n+1}^{(i)} \Delta\mathbf{x}_{n+1}^{(i)} = -\left[\mathbf{f}(\mathbf{x})\right]_{n+1}^{(i)} \tag{2}$$

where $\mathbf{f}(\mathbf{x})$ is the nonlinear system of equations, n is the current time step and i is the iteration number. While the original presentation of the multibody method ([1]) and a similar monolithic approach that uses the augmented Lagrangian [6] propose an approximate analytical expression for the tangent matrix, in this study it is formed numerically. This is to improve generality of the method, as in certain cases analytical methods can become too cumbersome to use in practice, and due to complicated forms of partial derivatives of force vector and pressure derivative equations.

Efficiency of the proposed method is evaluated against the method proposed by Naya et al. [6]. Results of a small scale case example are presented in Tab. 1, where AL refers to the monolithic scheme with the augmented Lagrangia and SR to the proposed method. As can be seen, use of the semi-recursive method in this context yields superior efficiency when compared to the previously presented method. The difference corresponds approximately to the difference in problem size.

Tab. 1: Relative CPU-times of the proposed method versus method presented in [6].

Δt	0.5 ms	1 ms	2 ms	5 ms
AL/SR	4.18	3.99	3.53	3.56

Energy balance of an example case is presented in Fig. 1. Good energy conservation properties can be observed, considered that the variations of approximately 1400 J in potential energy during the simulation. Thus, the demonstrated robustness [1] of the multibody method is preserved when combined with the hydraulic dynamics.

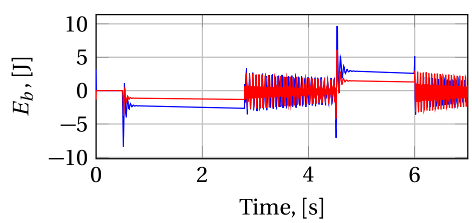

Fig. 1: Energy balance E_b with 8 ms step size (blue) and 4 ms step size (red).

A monolithic method, that uses the semi-recursive and lumped fluid methods, aimed for real-time simulation of multibody and hydraulic dynamics is presented in this study. The proposed method maintains the previously demonstrated robustness of the multibody method, and exhibits superior computational efficiency when compared to the equivalent approach presented in the literature

References

[1] J. Cuadrado, D. Dopico, M. Gonzalez, and M. Naya, "A combined penalty and recursive real-time formulation for multibody dynamics," *Journal of Mechanical Design, Transactions of the ASME*, vol. 126, no. 4, pp. 602–608, 2004.

[2] J. G. de Jalón and E. Bayo, *Kinematic and Dynamic Simulation of Multibody Systems: The Real-Time Challenge*. Springer-Verlag, New-York, 1994.

[3] E. Bayo and R. Ledesma, "Augmented Lagrangian and mass-orthogonal projection methods for constrained multibody dynamics," *Nonlinear Dynamics*, vol. 9, no. 1-2, pp. 113–130, 1996.

[4] J. Watton, *Fluid Power Systems: Modeling, Simulation, Analog and Microcomputer Control*. Prentice-Hall, Inc., 1989.

[5] J. Nokka, J.-H. Montonen, E. Bin Baharudin, P. Immonen, A. Rouvinen, L. Laurila, T. Lindh, A. Mikkola, J. Sopanen, and J. Pyrhönen, "Multi-body simulation based development environment for hybrid working machines," *International Review on Modelling and Simulations*, vol. 8, no. 4, pp. 466–476, 2015.

[6] M. Naya, J. Cuadrado, D. Dopico, and U. Lugris, "An efficient unified method for the combined simulation of multibody and hydraulic dynamics: Comparison with simplified and co-integration approaches," *Archive of Mechanical Engineering*, vol. 58, no. 2, pp. 223–243, 2011.