Limitations of finite difference methods in the computation of coupling forces prescribed by the Reynolds equation

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Thin viscous fluid films which carry load due to the effect of aero- or hydrodynamic lubrication are an important family of non-linear couplings in multi-body systems. These films can be found in radial and thrust hydrodynamic bearings, between pistons and liners and in linear and radial aerodynamic bearings. The pressure distribution in the film is governed by the Reynolds equation, a partial differential equation which can be derived from the Navier-Stokes equations [1].

The Reynolds equation can be solved analytically only in special cases [2] and in general has to be solved numerically. Currently, finite element (FEM) [3], finite volumes and finite difference (FDM) [1] methods are the most prominent numerical methods for solving this equation.

Almost all manufacturers of journal bearings provide data-sheets with design parameters. Stiffness and damping coefficients of the bearing at several speeds are commonly included in the sheet and thus a customer is able to perform a dynamic analysis of a rotor supported in the proposed bearings. However, some recent articles [4, 5] suggest that numerically-obtained functions which describe dependence of the coefficients on journal's speed are not continuous though the same functions obtained analytically are continuous and smooth [2]. Discontinuity of the above-mentioned numerically-obtained functions is studied in this paper, its root cause is identified and possible ways of mitigating its impact on results are discussed.

Distribution of hydrodynamic pressure $p = p(s, x, t)$ in a thin viscous fluid film in a generic journal bearing is assumed to be governed by the Reynolds equation in the form [1]

$$\frac{\partial}{\partial s}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial s}\right)+\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right)=6\,\omega_0\,r\,\frac{\partial h}{\partial s}+12\,\frac{\partial h}{\partial t}, \tag{1}$$

where s and x are circumferential and axial coordinates, respectively; they are shown in Fig. 1. $h = h(s, t)$ is a gap between the journal and the bearing shell, μ is the constant dynamic viscosity of the fluid film, r is the radius of the

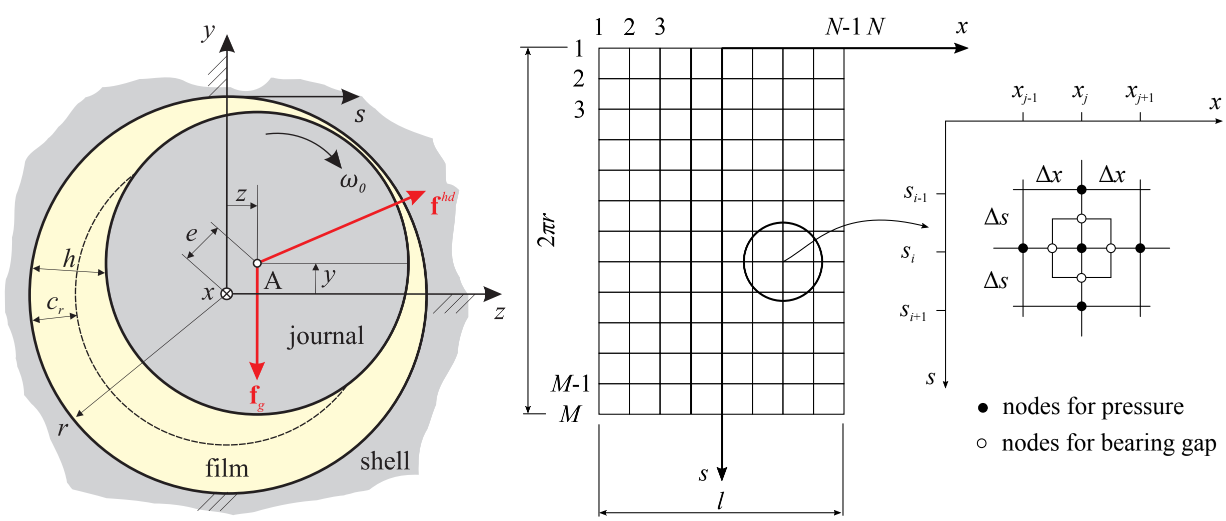

Fig. 1: A scheme of a generic journal bearing and a mesh for the finite difference method.

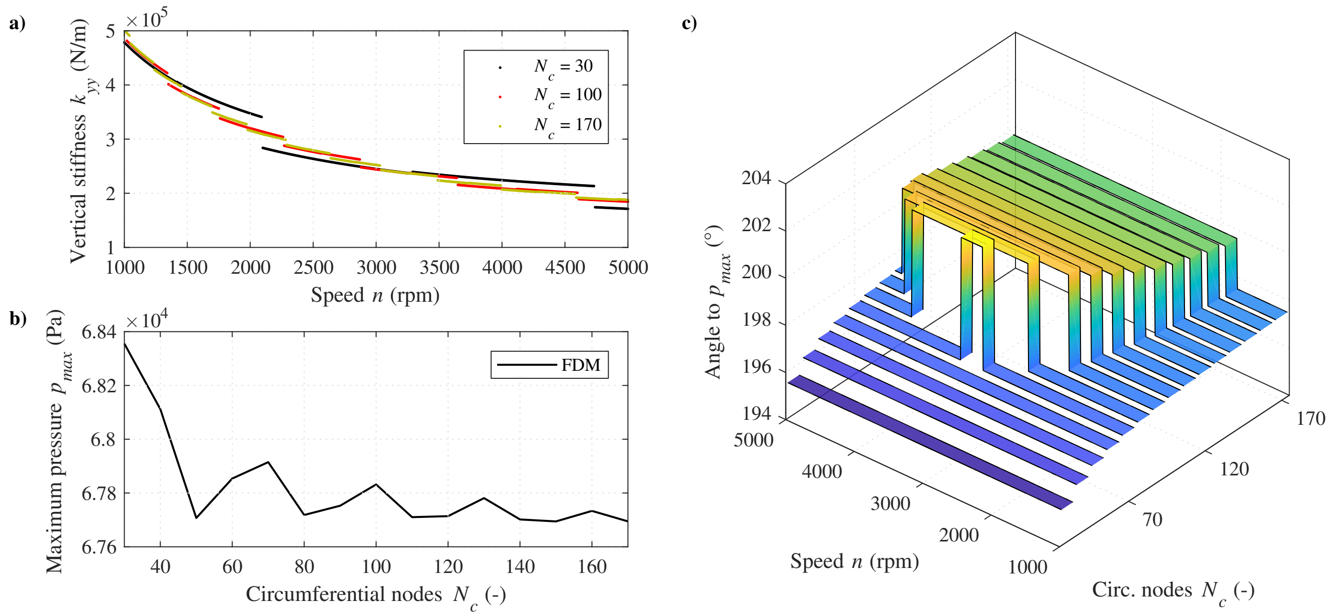

Fig. 2: Jump discontinuities in $k_{yy} = k_{yy}(\omega_0)$ a) are caused by the facts that both maximum pressure b) and angle to maximum pressure c) are dependant of the number of nodes in the circumferential direction of a computational mesh for the finite difference method.

bearing and ω_0 is the constant angular speed of the journal. Eq. (1) is solved using FDM described in [1]. Resulting hydrodynamic pressure p is then integrated over the bearing surface, which yields a vector of hydrodynamic forces $\mathbf{f}^{hd}$. If the journal is subjected to small displacement Δy, new vector $\mathbf{f}^{hd}_{\Delta y}$ can be obtained and stiffness coefficients k_{yy} and k_{yz} estimated. Remaining stiffness and damping coefficients can be calculated similarly.

As can be seen in Fig. 2a, the function $k_{yy} = k_{yy}(\omega_0)$ is discontinuous. Jumps in discontinuities are smaller if more circumferential nodes are present in a mesh for FDM. The number of nodes influences directly the maximum pressure in the film as shown in Fig. 2b and, more importantly, the angle to maximum pressure (Fig. 2c). This angle is important for the determination of direction of vector $\mathbf{f}^{hd}$ and is one of the main factors that cause the jump discontinuities. The obvious precaution against the jump discontinuities is using as many points in the circumferential direction of the mesh as possible.

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