Dynamic Behaviors of Composite Flexible Structure with Piezoelectric Actuators via Absolute Nodal Coordinate Formulation

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Composite flexible structures are widely applied on soft robots, which are combined with hard electric actuating material and flexible medium. Due to the geometrical and material nonlinearity features and the multi-element coupling problem with the actuators, the accurate dynamic modelling of the composite flexible structure is of great importance to the dynamic analysis for the soft robots. In this context, a plate element with Mooney-Rivlin hyperelastic constitutive model and a beam element with piezoelectric constitutive equations are developed based on the absolute nodal coordinate formulation (ANCF), and a composite beam-plate structure with piezoelectric actuator is established by applying the deformation compatibility conditions. Dynamic behaviors of the composite structure under piezoelectric actuation are investigated with various parameters.

In this study, a composite beam-plate structure is employed and is shown in Fig. 1. Beam actuator element is attached on the upper surface of the plate element, parallel to the side AB. The actuator is subjected to a certain voltage, and a structural deformation occurs due to the piezoelectric effect, which leads to a vibration of the composite structure.

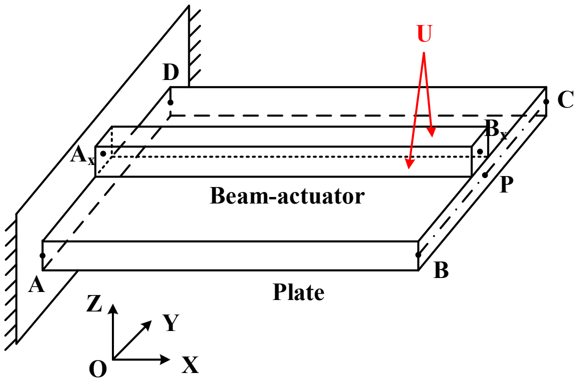


Fig.1: Composite cantilever structure with piezoelectric actuation

The deformation of the plate and beam element is described based on ANCF. The coordinates of an arbitrary point on the ANCF element can be described with the high-order shape function matrix $\mathbf{S}$ and the nodal coordinate of the element $\mathbf{e}$ as

$$\mathbf{r} = \mathbf{S} \cdot \mathbf{e} \tag{1}$$

In this study, $\mathbf{e}^P$ with 48 coordinates and $\mathbf{e}^B$ with 24 coordinates denote the absolute nodal coordinate of the plate and beam elements, respectively.

By employing the deformation compatibility conditions assumptions, the constrains equation at the interface of the composite elements can be written as

$$\mathbf{e}^B = \begin{bmatrix} \mathbf{T}_{G'}^B \\ \mathbf{T}_{F'}^B \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{T}_G^P \\ \mathbf{T}_F^P \end{bmatrix} \cdot \mathbf{e}^P = \mathbf{T}^{BP} \cdot \mathbf{e}^P \tag{2}$$

where $\mathbf{T}^{BP}$ is the transformation matrix of the nodal coordinate with a dimension of 24×48. With this transformation matrix, the nodal coordinate of the structure can be uniformed as the nodal coordinate of the plate element, and the continuity condition of the composite structure turns into class C^1 since the higher-order constraint equations are introduced.

The mass and stiffness matrix of the composite beam-plate structure can be calculated based on continuum mechanics theory and the transformation matrix as

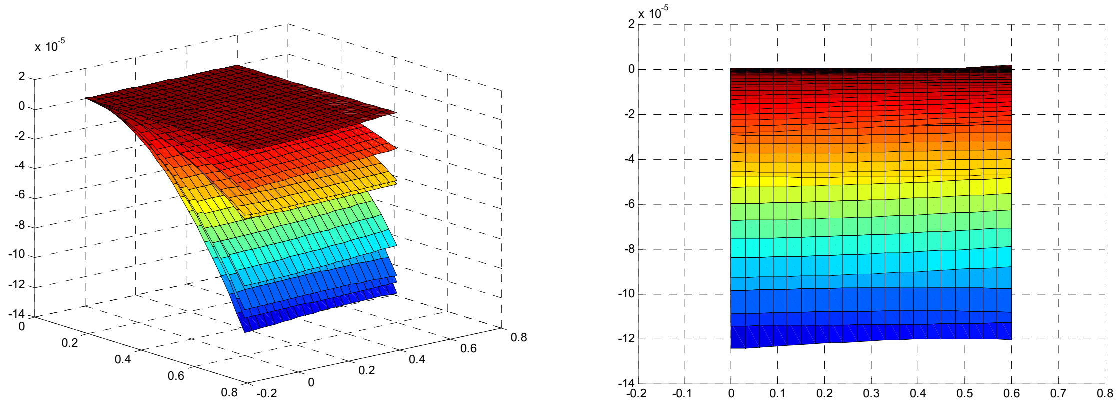
$$\begin{aligned}\mathbf{M}^{BP} &= \mathbf{M}^{P} + (\mathbf{T}^{BP})^{\mathrm{T}}\mathbf{M}^{B}\mathbf{T}^{BP} \\ \mathbf{K}^{BP} &= \mathbf{K}^{P} + (\mathbf{T}^{BP})^{\mathrm{T}}\mathbf{K}^{B}\mathbf{T}^{BP} + (\mathbf{T}^{BP})^{\mathrm{T}}\mathbf{K}^{E}\mathbf{T}^{BP}\end{aligned} \tag{3}$$

The dynamic equation of the composite structure can be defined based on Newton formulation as

$$\mathbf{M}^{BP}\ddot{\mathbf{e}} + \mathbf{K}^{BP}\mathbf{e} = \mathbf{Q}_a \tag{4}$$

here $\mathbf{Q}_a$ is the generalized external force, which can be formulated by using the shape function matrix and the vector of the external force as $\mathbf{Q}_a^{\mathrm{T}} = \mathbf{F}^{\mathrm{T}}\mathbf{S}$. Equation (4) can be solved by using the numerical computational methods, the displacement, velocity and acceleration of an arbitrary point on the structure can be calculated.

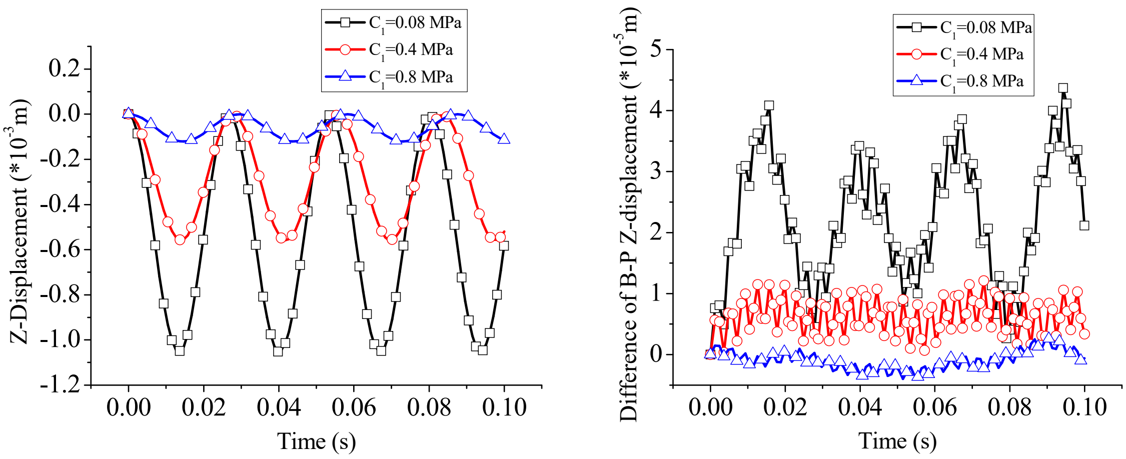
Fig.2 shows the deformation of the composite plate structure during vibration. Results indicate that the deformation of the composite structure varies at each time step, and there is a displacement difference of the endpoints and the mid-point C of side BC.



(a) X-Y-Z standard viewport (b) Y-Z plane viewport

Fig.2: Deformation of the composite plate structure at different time steps

Meanwhile, the kinematic and dynamic behaviors of the composite structure with different material parameters are discussed. The displacements of mid-point P and the displacement difference of node B and mid-point P with various parameters are shown in Figure 3. The results may provide the guidance for the actuation design and the dynamic analysis of soft robots.



(a) Z-Displacement of mid-point P (b) Difference of Z-Displacement of node B and point P

Fig.3: Correlation of displacement and time with different material parameters

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