Model-based pre-step stabilization method for non-iterative co-simulation

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Complex system simulations typically cover several domain-specific problems to be solved within tailored simulation tools. Analysis of the entire system behavior strictly requires faithful handling of the subsystem interactions - in terms of co-simulation - which crucially leads to efficient coupling strategies and related algorithms [1, 2]. Co-Simulation treats subsystems independently and data exchange is performed at predefined communication points in time for synchronization purposes. In connection to bidirectional dependencies between subsystems the resulting causality problem has to be solved by extrapolation of input quantities. In non-iterative (explicit) co-simulation the extrapolation of input quantities is directly associated to an estimation error, i.e. the discretization error. Especially for highly non-linear and stiff system dynamics, it is crucial to have more advanced (model-based) strategies in place. The aim of this paper is to present a model-based extension to an existing co-simulation algorithm, which minimizes the coupling error prior to (pre-step) the individual subsystem simulations.
The algorithm is based on time-variant linear subsystem approximations to increase co-simulation performance in terms of accuracy and stability. Three main steps are defined: (1) estimation of the exact (solution) output utilizing an introduced *Error Differential Equation*, (2) model-based extrapolation of output quantities by model-based incorporation of cross-couplings and (3) optimization of the final subsystem inputs for the next macro-time step (pre-step) accordingly.

The three steps are based on the linearisation of the individual subsystems S_i, according to the subsystems inputs u_i and the related subsystem simulation results $\bar{y}_i$

$$\dot{\bar{y}}_i = \frac{\partial S_i}{\partial \bar{y}} \bar{y}_i + \frac{\partial S_i}{\partial u} u_i, \tag{1}$$

for $i = 1, \ldots, N$. $\frac{\partial S_i}{\partial \bar{y}}$, $\frac{\partial S_i}{\partial u}$ stands for the appropriate input-output sensitivities, where N denotes the number of subsystems. To keep the notation simple, the main steps are exemplarily shown for a fully coupled system of two subsystems. The first step of the algorithm is to solve, the introduced *Error Differential Equation*

$$\begin{pmatrix} \dot{\delta}_1 \\ \dot{\delta}_2 \end{pmatrix} = \begin{pmatrix} \frac{\partial S_1}{\partial \bar{y}} & \frac{\partial S_1}{\partial u} \\ \frac{\partial S_2}{\partial u} & \frac{\partial S_2}{\partial \bar{y}} \end{pmatrix} \cdot \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} + \begin{pmatrix} \frac{\partial S_1}{\partial u} & 0 \\ 0 & \frac{\partial S_2}{\partial u} \end{pmatrix} \cdot \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}, \tag{2}$$

which describes (1^{st} order approx.) the deviation of the simulation result $\bar{y}$ to the exact solution y, with

$$\begin{aligned} \delta_1 &= y_1 - \bar{y}_1, \qquad & \delta_2 &= y_2 - \bar{y}_2, \\ \varepsilon_2 &= \bar{y}_2 - u_1, \qquad & \varepsilon_2 &= \bar{y}_1 - u_2. \end{aligned}$$

In a second step the (approx.) exact solution y is model-based extrapolated, with the use of

$$\begin{pmatrix} \dot{y}_{e_1} \\ \dot{y}_{e_2} \end{pmatrix} = \begin{pmatrix} \frac{\partial S_1}{\partial \bar{y}} & \frac{\partial S_1}{\partial u} \\ \frac{\partial S_2}{\partial u} & \frac{\partial S_2}{\partial \bar{y}} \end{pmatrix} \cdot \begin{pmatrix} y_{e_1} \\ y_{e_2} \end{pmatrix}, \qquad \begin{pmatrix} y_{e_1}(T_k) \\ y_{e_2}(T_k) \end{pmatrix} = \begin{pmatrix} y_1(T_k) \\ y_2(T_k) \end{pmatrix}, \tag{3}$$

where y_e denote the extrapolation of y, in the next macro time step. The result $y(T_k)$ of the preceeding step represents the initial values. Due to model-based extrapolation cross coupling effects (off-diagonal entries) are

considered. In order to optimize the coupling error a minimization of $\alpha := |y_e - \bar{y}|$ over the next macro time step is performed. This leads, in the case of zero order hold extrapolation for the input u_i, to

$$\phi_{B_i} \cdot u_i = (y_{e_i} - \phi_{A_i} \bar{y}_i), \tag{4}$$

where ϕ_{A_i} and ϕ_{B_i} denotes the appropriate transition matrices of the subsystem S_i. In the case, that ϕ_{B_i} is singular, the pseudo inverse should be used to compute u_i. If the number of inputs is higher than the number of outputs per subsystem, extra conditions are needed to bring ϕ_{B_i} into a regular and quadratic structure. Otherwise, if there are more outputs in a subsystem than inputs a least square solution is the preferred way to compute u_i.

Stiff Non-Linear Multibody System Example

For demonstration, a recently analyzed non-linear multibody system example, a mathematical pendulum coupled to an oscillator, representing a stiff system, has been chosen [3]. The linearisation is based on the analytical computation of the input-output sensitivities and therefore $\partial S_i/\partial u$ and $\partial S_i/\partial \bar{y}$ are updated at every communication point in time. The benefit in accuracy of the introduced algorithm compared to the classical parallel zero order hold (ZOH) coupling algorithm [3] and the classical nearly energy preserving coupling element (NEPCE) algorithm [4], is depicted in Figure 1. In contrast to the global error estimation in [3], the coupling error is mitigated by the introduced algorithm for $H < 10^{-2}$. Meaning, with the use of the introduced algorithm it is possible, to enlarge the macro step-size and therefore the communication effort between the subsystems is reduced, improving the performance of the co-simulation.

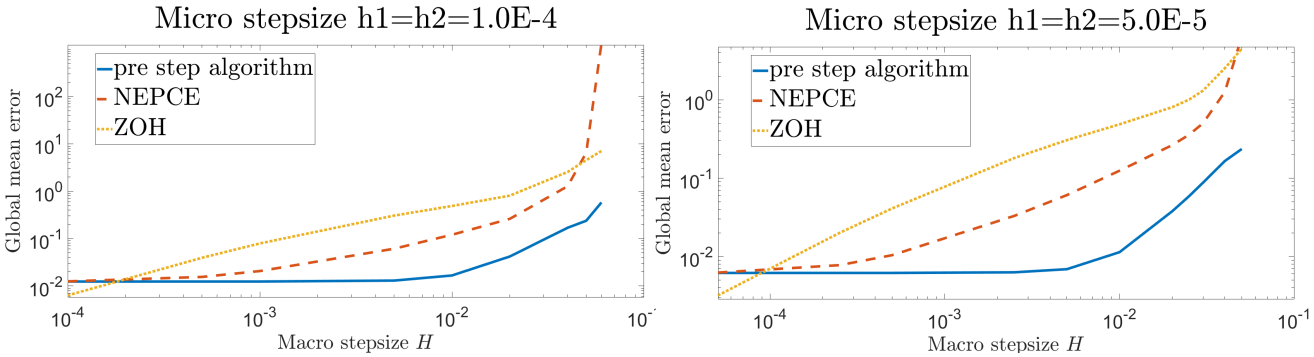

Fig. 1: Mean error, according to the monolithic solution of the linearisied problem, of the introduced pre step algorithm compared to NEPCE and ZOH algorithms. parameters equal to [3]; $K = 10^3 N/m$; $d = 10\, Ns/m$ for $t \in [0,1]$; $\lambda_{max}/\lambda_{min} \approx 10^5$.

Conclusion

For efficient handling of stiff system model-based numerical schemes have to be applied. In contrast to existing approaches, the proposed algorithm is based on a 1^{st} order approximation of the exact solution and, in addition, performs an optimization-based pre-step correction, resulting in a significant performance improvement in terms of accuracy and stability. Enlargement of the macro step-size ($\approx$ 100 times) yields overall co-simulation improvement. Future work focusses on identification of sensitivities, handling of DAE Systems and industrial applications.

References

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